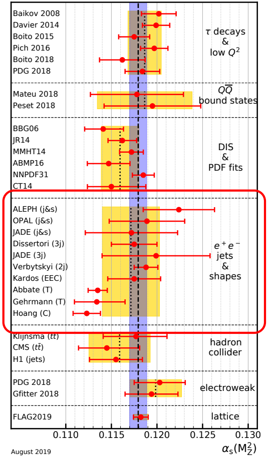
Determination of the strong coupling beyond NNLO using event shape averages

Gábor Somogyi
Wigner Research Centre for Physics

based on A. Kardos, GS, A. Verbytskyi, Eur. Phys. J. C 81 (2021) 4, 292
[arXiv:2009.00281 [hep-ph]]

16 September 2021

The strong coupling from e^+e^- annihilation



[P. A. Zyla et al. (Particle Data Group), Prog. Theor. Exp. Phys. 2020, 083C01 (2020) and 2021 update]

Why α_s in e^+e^-?

- $\alpha_s(M_Z)$ is known with $\sim 0.8\%$ precision (lattice)
- The e^+e^- jets & shapes sub-field alone gives $\sim 2.6\%$ uncertainty: large spread between measurements
- **Can $\sim$ 1% precision be achieved**?

What are the differences?

- **Hadronization modeling**: Monte Carlo or analytic
- Perturbative order: fixed order NNLO to N^{3}LO + resummation NLL to N^{3}LL
- Type of observable used: event shapes or jet rates

How best to improve?

Overview: issues and this work

The present situation raises some issues:

- No new data foreseen in the near future, so would including **more perturbative orders** (fixed order and/or resummation) improve precision **without any new data**?
- If not, what are the **limiting factors** for precision in future QCD studies?
- What should be done to **eliminate** those factors?

$$\Downarrow$$

To address these questions we perform a state-of-the-art perturbative QCD (pQCD) analysis with

- estimations of unknown higher order pQCD corrections from data: focus on event shape averages (small number of perturbative coefficients to fit)
- hadronization corrections obtained using both modern Monte Carlo tools as well as analytic models extended to higher perturbative orders

Event shape moments: theoretical description

The n-th moment of an event shape O is defined by

$$\langle O^n \rangle = \frac{1}{\sigma_{\rm tot}} \int_{O_{\rm min}}^{O_{\rm max}} O^n \frac{{\rm d}\sigma(O)}{{\rm d}O} {\rm d}O$$

Fixed-order predictions up to and including α_s^4 terms read

$$\langle O^n \rangle = \frac{\alpha_s(Q)}{2\pi} A^{\langle O^n \rangle} + \left(\frac{\alpha_s(Q)}{2\pi}\right)^2 B^{\langle O^n \rangle} + \left(\frac{\alpha_s(Q)}{2\pi}\right)^3 C^{\langle O^n \rangle} + \left(\frac{\alpha_s(Q)}{2\pi}\right)^4 D^{\langle O^n \rangle} + \mathcal{O}(\alpha_s^5)$$

- First three coefficients ($A^{\langle O^n \rangle}$, $B^{\langle O^n \rangle}$ and $C^{\langle O^n \rangle}$) known for some time

  [Gehrmann-De Ridder et al., JHEP **05** (2009) 106 (GGGH), Weinzierl, Phys. Rev. D **80** (2009) 094018] (SW)
- Recomputed for this study using CoLoRFulNNLO $\Rightarrow$ very good numerical precision

  [Del Duca et al., Phys. Rev. D **94** (2016) no.7, 074019]
- b-mass corrections from Zbb4: note only NLO

  [Nason, Oleari, Phys. Lett. B **407**, 57 (1997)]

$$A^{\langle O^n \rangle} = (1 - r_b(Q)) A^{\langle O^n \rangle}_{m_b=0} + r_b(Q) A^{\langle O^n \rangle}_{m_b\neq 0}$$

$$B^{\langle O^n \rangle} = (1 - r_b(Q)) B^{\langle O^n \rangle}_{m_b=0} + r_b(Q) B^{\langle O^n \rangle}_{m_b\neq 0}$$

where r_b is the fraction of b-quark events

$$r_b(Q) = \frac{\sigma_{m_b\neq 0}(e^+e^- \to b\bar{b})}{\sigma_{m_b\neq 0}(e^+e^- \to {\rm hadrons})}$$

Event shape averages: predictions at NNLO and beyond

We focus on averages of the C-parameter $\langle C^1 \rangle$ and one minus thrust $\langle (1-T)^1 \rangle$

- abundance of available measurements (see below)
- avoid correlations between various moments (not reported by most measurements)

Fixed-order predictions at scale $Q = m_Z$ for the perturbative coefficients [normalized to the leading order cross section $\sigma_0(e^+e^- \to \text{hadrons})$]

Coefficient	This work	Analytic	GGGH	SW
$A_0^{\langle (1-T)^1 \rangle}$	2.1034(1)	2.10347	2.1035	2.10344(3)
$B_0^{\langle (1-T)^1 \rangle}$	44.995(1)		44.999(2)	44.999(5)
$C_0^{\langle (1-T)^1 \rangle}$	979.6(6)		867(21)	1100(30)
$A_0^{\langle C^1 \rangle}$	8.6332(5)	8.63789	8.6379	8.6378(1)
$B_0^{\langle C^1 \rangle}$	172.834(5)	172.859	172.778(7)	172.8(3)
$C_0^{\langle C^1 \rangle}$	3525(3)		3212(89)	4200(100)

[Gehrmann-De Ridder et al., JHEP **05** (2009) 106 (GGGH), Weinzierl, Phys. Rev. D **80** (2009) 094018] (SW)

We extract $D^{\langle (1-T)^1 \rangle}$ and $D^{\langle C^1 \rangle}$ from data together with $\alpha_s(M_Z)$ in the analysis.

Importantly, the main point of extracting the N^{3}LO coefficients $D^{\langle (1-T)^1 \rangle}$ and $D^{\langle C^1 \rangle}$ from data is **not** to get an accurate determination of these quantities.

Rather, it is to model them as best as possible in order to be able to **assess the impact** of including terms beyond NNLO in the extraction of the strong coupling in the absence of an actual calculation of those terms.

Modeling non-perturbative corrections

The modeling of non-perturbative corrections is essential to perform a meaningful comparison of predictions with data.

To basic approaches

1. **Monte Carlo (MC) hadronization**: extract hadronization corrections from Monte Carlo simulations.

   Issue: the parton level of an MC simulation is not equivalent to a fixed-order calculation.

2. **Analytic hadronization**: use analytic models to describe the effects of hadronization on observables.

   Issue: systematics are difficult to control.

$$\Downarrow$$

Apply both approaches and examine the impact of the choice on the extracted value of the strong coupling.

Monte Carlo hadronization

Hadronization corrections obtained using state-of-the-art MC event generators:
$e^+e^- \to Z/\gamma \to 2, 3, 4, 5$ parton processes generated using `MadGraph5` and `OpenLoops`, 2-parton final state at NLO.

To study hadronization systematics, we employ different setups:

- **Default setup "H^L": `Herwig7.2.0` with Lund fragmentation model**
- Setup for systematics "H^C": `Herwig7.2.0` with cluster fragmentation model
- Setup for cross-checks "S^C": `Sherpa2.2.8` with cluster fragmentation model

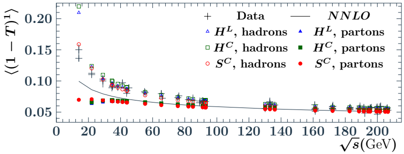
Hadronization corrections are ratios of observables calculated from MC generated events at hadron and parton levels.

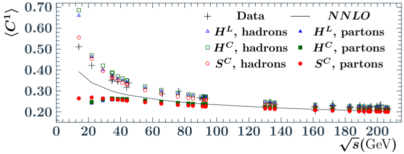
To account for the presence of a shower cut-off scale $Q_0 \approx \mathcal{O}(1\,\text{GeV})$ in MC generators, predictions were computed with several values of Q_0 and extrapolated to $Q_0 \to 0$ GeV.

$$\langle O^n \rangle_{\text{corrected}} = \langle O^n \rangle_{\text{theory}} \times \frac{\langle O^n \rangle_{\text{MC hadrons, } Q_0=0\text{ GeV}}}{\langle O^n \rangle_{\text{MC partons, } Q_0=0\text{ GeV}}}$$

Monte Carlo hadronization

Data and predictions by MC event generators extrapolated to $Q_0 \to 0$ GeV.





- Hadron and parton level MC predictions provide **reasonable descriptions** of data and NNLO theory for wide range of energy
- **Non-physical** behaviour of MC parton level results for small $\sqrt{s}$: $\langle O^n \rangle$ increases with energy

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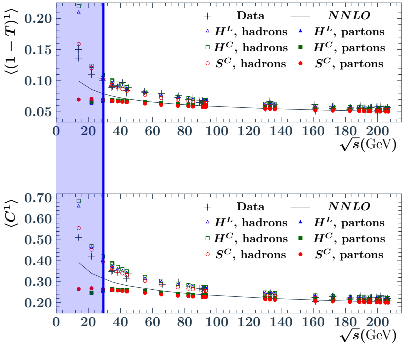


- Hadron and parton level MC predictions provide **reasonable descriptions** of data and NNLO theory for wide range of energy
- **Non-physical** behaviour of MC parton level results for small $\sqrt{s}$: $\langle O^n \rangle$ increases with energy

$\Downarrow$

- **Exclude measurements** with $\sqrt{s} < 29$ GeV
- Weaker criterion than requiring that MC matches data well, but retains as much data as possible

Analytic hadronization

Dispersive model of analytic hadronization corrections for event shapes: hadronization corrections simply shift the perturbative event shape averages

$$\langle O^1 \rangle_{\text{hadrons}} = \langle O^1 \rangle_{\text{partons}} + a_O \mathcal{P}$$

- the a_O are observable-specific constants, e.g., $a_{1-T} = 2$ and $a_C = 3\pi$
- the power correction $\mathcal{P}$ is universal

We must compute $\mathcal{P}$ at $\mathcal{O}(\alpha_s^4)$ accuracy. Ingredients of the computation are

- The running of the strong coupling in the $\overline{\text{MS}}$ scheme
- The relation between the effective soft coupling in the Catani–Marchesini–Webber (CMW) scheme α_s^{CMW} and the strong coupling defined in the $\overline{\text{MS}}$ scheme α_s

$$\alpha_S^{CMW} = \alpha_S \left[1 + \frac{\alpha_S}{2\pi} K + \left(\frac{\alpha_S}{2\pi} \right)^2 L + \left(\frac{\alpha_S}{2\pi} \right)^3 M + \mathcal{O}(\alpha_S^4) \right]$$

- K is simply the one-loop cusp anomalous dimension
- L and M can be computed once the effective soft coupling is explicitly defined $\Rightarrow$ several proposals in the literature beyond NLL, so L and M are "scheme-dependent"

The power correction

The power correction at $\mathcal{O}(\alpha_s^4)$ accuracy reads

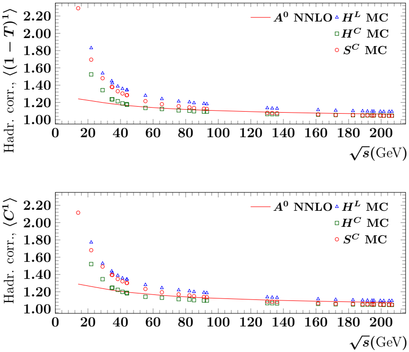
$$\mathcal{P}(\alpha_S, Q, \alpha_0) = \frac{4C_F}{\pi^2}\,\mathcal{M} \times \frac{\mu_I}{Q} \times \Bigg\{\alpha_0(\mu_I) - \Bigg[\alpha_S(\mu_R) + \left(K + \beta_0\left(1 + \ln\frac{\mu_R}{\mu_I}\right)\right)\frac{\alpha_S^2(\mu_R)}{2\pi}$$
$$+ \left(2L + (4\beta_0(\beta_0 + K) + \beta_1)\left(1 + \ln\frac{\mu_R}{\mu_I}\right) + 2\beta_0^2\ln^2\frac{\mu_R}{\mu_I}\right)\frac{\alpha_S^3(\mu_R)}{8\pi^2}$$
$$+ \Bigg(4M + (2\beta_0(12\beta_0(\beta_0 + K) + 5\beta_1) + \beta_2 + 4\beta_1 K + 12\beta_0 L)\left(1 + \ln\frac{\mu_R}{\mu_I}\right)$$
$$+ \beta_0(12\beta_0(\beta_0 + K) + 5\beta_1)\ln^2\frac{\mu_R}{\mu_I} + 4\beta_0^3\ln^3\frac{\mu_R}{\mu_I}\Bigg)\frac{\alpha_S^4(\mu_R)}{32\pi^3}\Bigg]\Bigg\}$$

- $\mathcal{M}$ is the so-called Milan factor with estimated value $\mathcal{M}_{\text{est.}} \pm \delta\mathcal{M}_{\text{est.}} = 1.49 \pm 0.30$.
- μ_I is the scale where the perturbative and non-perturbative couplings are matched. Following the usual choice, we set $\mu_I = 2\,\text{GeV}$.
- $\alpha_0(\mu_I)$ corresponds to the first moment of the effective soft coupling below the scale μ_I and is a **non-perturbative parameter** of the model

$$\alpha_0(\mu_I) = \frac{1}{\mu_I}\int_0^{\mu_I} \mathrm{d}\mu\,\alpha_s^{CMW}(\mu)$$

Hadronization correction factors

Ratios of hadron-level to parton-level predictions



- Analytic hadronization "schemes": the L and M coefficients entering the power correction $\mathcal{P}$ depend on the precise definition of α_s^{CMW} beyond NLL $\Rightarrow$ different "schemes": A^0, A^T and A^{cusp}

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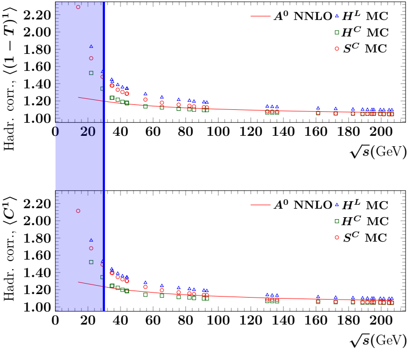


- Analytic hadronization "schemes": the L and M coefficients entering the power correction $\mathcal{P}$ depend on the precise definition of α_s^{CMW} beyond NLL $\Rightarrow$ different "schemes": A^0, A^T and A^{cusp}
- Recall measurements with $\sqrt{s} < 29$ GeV are excluded.
- Weaker criterion than requiring that sub-leading power corrections are small.
- Serves to highlight the discrepancies between MC and analytic models where hadronization effects are most pronounced (low energies).

Data

Combined analysis using 20+ datasets and a **wide range of energies**: $\sqrt{s} = 29$–206 GeV

	Measured		Used	
Source	Observables	Points, $\sqrt{s}$ range (GeV)	Observables	Points, $\sqrt{s}$ range (GeV)
ALEPH	$\langle(1-T)^1\rangle$	1,[133]	$\langle(1-T)^1\rangle$	1,[133]
ALEPH	$\langle(1-T)^1\rangle$	1,[91]	$\langle(1-T)^1\rangle$	1,[91]
ALEPH	$\langle(1-T)^1\rangle$	9,[91, 206]	$\langle(1-T)^1\rangle$	9,[91, 206]
AMY	$\langle(1-T)^1\rangle$	1,[55]	$\langle(1-T)^1\rangle$	1,[55]
DELPHI	$\langle(1-T)^{1,2,3}\rangle$	15,[91, 183]	$\langle(1-T)^1\rangle$	5,[91, 183]
DELPHI	$\langle(1-T)^1\rangle$	15,[45, 202]	$\langle(1-T)^1\rangle$	11,[45, 202]
HRS	$\langle(1-T)^1\rangle$	1,[29]	$\langle(1-T)^1\rangle$	1,[29]
JADE	$\langle(1-T)^{1,2,3,4,5}\rangle$	30,[14, 43]	$\langle(1-T)^1\rangle$	4,[34, 43]
L3	$\langle(1-T)^1\rangle$	1,[91]	$\langle(1-T)^1\rangle$	1,[91]
L3	$\langle(1-T)^{1,2}\rangle$	30,[41, 206]	$\langle(1-T)^1\rangle$	15,[41, 206]
MARK	$\langle(1-T)^1\rangle$	1,[89]	$\langle(1-T)^1\rangle$	1,[89]
MARK	$\langle(1-T)^1\rangle$	1,[29]	$\langle(1-T)^1\rangle$	1,[29]
MARKII	$\langle(1-T)^1\rangle$	1,[89]	$\langle(1-T)^1\rangle$	1,[89]
OPAL	$\langle(1-T)^{1,2,3,4,5}\rangle$	60,[91, 206]	$\langle(1-T)^1\rangle$	12,[91, 206]
TASSO	$\langle(1-T)^1\rangle$	4,[14, 44]	$\langle(1-T)^1\rangle$	2,[35, 44]
ALEPH	$\langle C^1\rangle$	1,[91]	$\langle C^1\rangle$	1,[91]
DELPHI	$\langle C^1\rangle$	15,[45, 202]	$\langle C^1\rangle$	11,[45, 202]
DELPHI	$\langle C^{1,2,3}\rangle$	12,[133, 183]	$\langle C^1\rangle$	4,[133, 183]
JADE	$\langle C^{1,2,3,4,5}\rangle$	30,[14, 43]	$\langle C^1\rangle$	4,[34, 43]
L3	$\langle C^1\rangle$	1,[91]	$\langle C^1\rangle$	1,[91]
L3	$\langle C^{1,2}\rangle$	18,[130, 206]	$\langle C^1\rangle$	9,[130, 206]
OPAL	$\langle C^{1,2,3,4,5}\rangle$	60,[91, 206]	$\langle C^1\rangle$	12,[91, 206]

Fit procedure

Values of α_s determined using optimization procedures in `MINUIT2`

$$\chi^2(\alpha_S) = \sum_i^{\text{all data sets}} \chi_i^2(\alpha_S)$$

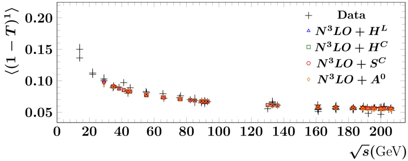
where $\chi_i^2(\alpha_S)$ for data set i is

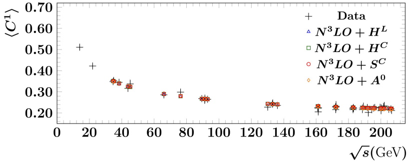
$$\chi_i^2(\alpha_S) = (\vec{D} - \vec{P}(\alpha_S))V^{-1}(\vec{D} - \vec{P}(\alpha_S))^T$$

- $\vec{D}$: vector of data points
- $\vec{P}(\alpha_S)$: vector of calculated predictions
- V: the covariance matrix of $\vec{D}$ (diagonal, stat. and syst. uncertainties added in quadrature for every measurement)

Fit results

Results of the fits at N^{3}LO vs. data. In addition to $\alpha_s(M_Z)$, we fit also

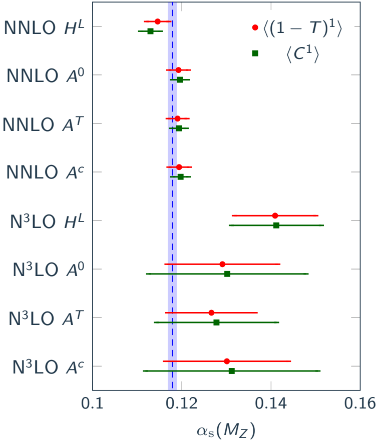




- the $\mathcal{O}(\alpha_s^4)$ perturbative coefficient $D^{\langle O^n\rangle}$ (in N^{3}LO fits)
- the non-perturbative parameter $\alpha_0(2\,\mathrm{GeV})$ (when using the analytic hadronization model)
- the Milan factor $\mathcal{M}$, in order to include the uncertainty on its theoretical value consistently (constrained fit)
- note the dependence on analytic hadronization scheme is mild so only the result for the A^0 scheme is shown

Results: $\alpha_s(M_Z)$

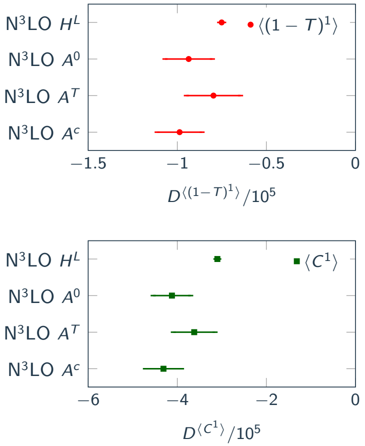
The extractions of $\alpha_s(M_Z)$ from $\langle(1-T)^1\rangle$ and $\langle C^1\rangle$ data



- Good agreement between fits to $\langle(1-T)^1\rangle$ and $\langle C^1\rangle$ data both at NNLO and N^3LO $\Rightarrow$ internal consistency of extraction procedure
- Analytic hadronization "scheme-dependence" is mild.
- **Large discrepancy between results obtained with MC and analytic hadronization models** both at NNLO and N^3LO $\Rightarrow$ suggests that the discrepancy has a fundamental origin and would hold even with exact N^3LO predictions.
- **Better understanding of hadronization is key.**

Results: $D^{\langle O^n \rangle}$

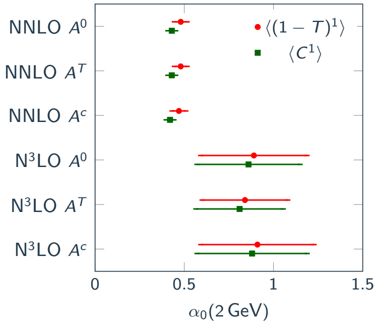
The extractions of the $\mathcal{O}(\alpha_s^4)$ perturbative coefficients $D^{\langle (1-T)^1 \rangle}$ and $D^{\langle C^1 \rangle}$ from data



- Extracted values of the perturbative coefficients show reasonable agreement for both observables between fits using MC and analytic hadronization models $\Rightarrow$ demonstrates the viability of extracting higher-order coefficients from data
- The amount and consistency of current data is an issue, would need large amounts of consistent data, e.g., from FCC-*ee* or CEPC.
- Precise high-energy data would be especially valuable.

Results: $\alpha_0(2\,\text{GeV})$

The extractions of the non-perturbative parameter $\alpha_0(2\,\text{GeV})$ from $\langle(1-T)^1\rangle$ and $\langle C^1\rangle$ data



- Recall this parameter is "scheme-dependent", so its values in different schemes should not be directly compared. Nevertheless, **the choice of scheme has only a small numerical impact**.
- Values extracted from $\langle(1-T)^1\rangle$ and $\langle C^1\rangle$ data agree well with each other both at NNLO and N^{3}LO
- Rather large uncertainties at N^{3}LO primarily due to insufficient amount and quality of data as well as the extraction method itself.

Discussion

The aim of the analysis was to assess the factors that will determine the precision of QCD analyses of e^+e^- data once theoretical predictions at $\mathcal{O}(\alpha_s^4)$ accuracy become available.

To do this, we have performed an extraction of $\alpha_s(M_Z)$ from the averages of event shapes $\langle(1-T)^1\rangle$ and $\langle C^1\rangle$.

- Using NNLO theory and analytic hadronization models, the obtained results are consistent with the last world average $\alpha_s(M_Z)_{\mathrm{PDG2020}} = 0.1179 \pm 0.0010$.
- We considered a method of extracting $\alpha_s(M_Z)$ at N^3LO by estimating the missing $\mathcal{O}(\alpha_s^4)$ perturbative coefficient from data. The values of $\alpha_s(M_Z)$ obtained in this way are compatible with the last world average, within somewhat large uncertainties, e.g.,

$$\alpha_s(M_Z)^{N^3LO+A^0} = 0.12911 \pm 0.00177(exp.) \pm 0.0123(scale)$$

- Both MC and analytic hadronization models were used, the latter being extended to $\mathcal{O}(\alpha_s^4)$ for the first time.
- The comparison of results obtained with MC and analytic hadronization suggests that future extractions of $\alpha_s(M_Z)$ will be strongly affected by the modeling of hadronization effects.

Discussion

Improving the perturbative predictions is clearly important

- beyond NNLO/NLL accuracy for event shapes
- mass corrections (finite m_b) beyond NLO
- mixed EW$\times$QCD corrections

But the elephant in the room: hadronization modeling

- naively going to higher energies helps: hadr. corr. $\sim 1/Q$, however...
- the energy of foreseen machines (FCC-*ee*, CEPC) is not orders of magnitude larger than LEP
- moreover, going up in energy there is non-trivial interplay between smaller hadronization corrections but larger background and much smaller luminosity

Discussion

Bottom line: **need better MC's + hadronization models/calibration** in e^+e^-

In a perfect world

- Parton showers with NNLL logarithmic accuracy matched to NNLO
- Hadronization models calibrated from scratch with many different observables, since current models were tuned using MC's with lower accuracy

Alternatively

- Need a (much) more refined analytical understanding of non-perturbative corrections, for recent advances see e.g.,

  [Luisoni, Monni, Salam, Eur. Phys. J. C **81** (2021) 2, 158, Caola et al., arXiv:2108.08897 [hep-ph]]
- Look for better observables with smaller hadronization corrections, e.g., groomed event shapes

  [Baron, Marzani, Theeuwes, JHEP **08** (2018) 105, Kardos, Larkoski, Trócsányi, Phys. Lett. B **809** (2020) 135704]

Conclusions

So where do we stand?

- No new data foreseen in the near future, so would including **more perturbative orders** (fixed order and/or resummation) improve precision **without any new data**?

  Not by itself. More perturbative orders alone are not likely to dramatically improve the precision of strong coupling extractions from existing data.

- If not, what are the **limiting factors** for precision in future QCD studies?

  Main limiting factors are: systematics related to the estimation of **hadronization corrections** as well as the quality and consistency of current data.

- What should be done to **eliminate** those factors?

  In addition to advancing the perturbative predictions, we **must seriously refine our understanding/modeling of non-perturbative effects**. This would be aided greatly by dedicated low-energy (below the Z-peak) measurements at future e^+e^- facilities.

Thank you for your attention!

Backup slides

The dispersive model of analytic hadronization corrections for event shapes gives

$$\frac{\mathrm{d}\sigma_{\text{hadrons}}(O)}{\mathrm{d}O} = \frac{\mathrm{d}\sigma_{\text{partons}}(O - a_O\mathcal{P})}{\mathrm{d}O}$$

We then obtain $\langle O^1\rangle_{\text{hadrons}} = \langle O^1\rangle_{\text{partons}} + a_O\mathcal{P}$ under the **assumptions**:

- the a_O are observable-specific **constants**

  Issue: a_O have been computed in the two-jet limit, but they actually depend on the value of O

  [Luisoni, Monni, Salam, Eur. Phys. J. C **81** (2021) 2, 158, Caola et al., arXiv:2108.08897 [hep-ph]]

- the power correction $\mathcal{P}$ is **universal**

$$\mathcal{P}(\alpha_\mathrm{s}, Q, \alpha_0) = \frac{4C_F}{\pi^2}\mathcal{M} \times \frac{\mu_I}{Q} \times \left\{\alpha_0(\mu_I) - \alpha_S + \mathcal{O}(\alpha_\mathrm{s}^2)\right\}$$

  Issue: non-inclusive corrections, e.g., those parametrized by the Milan factor $\mathcal{M}$ may not be universal beyond NLO

The validity of these model assumptions should be investigated.

Effective soft coupling schemes

The Catani–Marchesini–Webber soft coupling at NLL (α_s is the strong coupling in the $\overline{\text{MS}}$ scheme, $C_q = C_F,\ C_g = C_A$)

$$\mathcal{A}_i^{CMW}(\alpha_s) = C_i \frac{\alpha_s^{CMW}}{\pi} = C_i \frac{\alpha_s^{CMW}}{\pi} \left(1 + \frac{\alpha_s}{2\pi} K\right)$$

Proposals for definitions beyond NLL

$$\mathcal{A}_{T,i}(\alpha_s) = \frac{1}{2}\mu^2 \int_0^\infty dm_T^2 dk_T^2 \delta(\mu^2 - k_T^2) w_i(k)$$
$$\mathcal{A}_{0,i}(\alpha_s) = \frac{1}{2}\mu^2 \int_0^\infty dm_T^2 dk_T^2 \delta(\mu^2 - m_T^2) w_i(k)$$

where $w_i(k)$ is called the web function, it gives the "probability" of correlated emission (including the corresponding virtual corrections) of an arbitrary number of soft partons with total momentum k.

[Catani, De Florian and Grazzini, Eur. Phys. J. C **79**, 685 (2019), Banfi, El-Menoufi and P.F. Monni JHEP **01**, 083 (2019)]

Effective soft coupling schemes

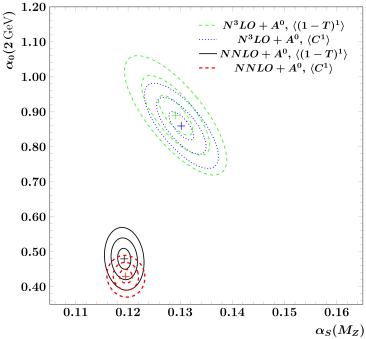
Given these definitions, the expansion of α_s^{CMW} in terms of α_s, and hence L and M, can in principle be computed (note in each scheme K is the one-loop cusp anomalous dimension)

$$(\alpha_s^{CMW})_{\text{scheme}} = \alpha_s \left[1 + \frac{\alpha_s}{2\pi} K + \left(\frac{\alpha_s}{2\pi}\right)^2 L_{\text{scheme}} + \left(\frac{\alpha_s}{2\pi}\right)^3 M_{\text{scheme}} + \mathcal{O}(\alpha_s^4)\right]$$

- A^0 scheme: L and M computed from $\mathcal{A}_{0,i}$
- A^T scheme: L computed from $\mathcal{A}_{T,i}$, but the complete expression for M is missing in this scheme, hence we set $M_T = M_0$
- A^{cusp} scheme: L and M are simply the two- and three-loop cusp anomalous dimensions

Correlations: $\alpha_s(M_Z)$ vs. $\alpha_0(2\,\text{GeV})$

Correlations between $\alpha_s(M_Z)$ and $\alpha_0(2\,\text{GeV})$



- contours correspond to 1-, 2- and 3 standard deviations obtained in the fit
- systematic uncertainties not included