Soft-Dropped Observables with CoLoRFuLNNLO

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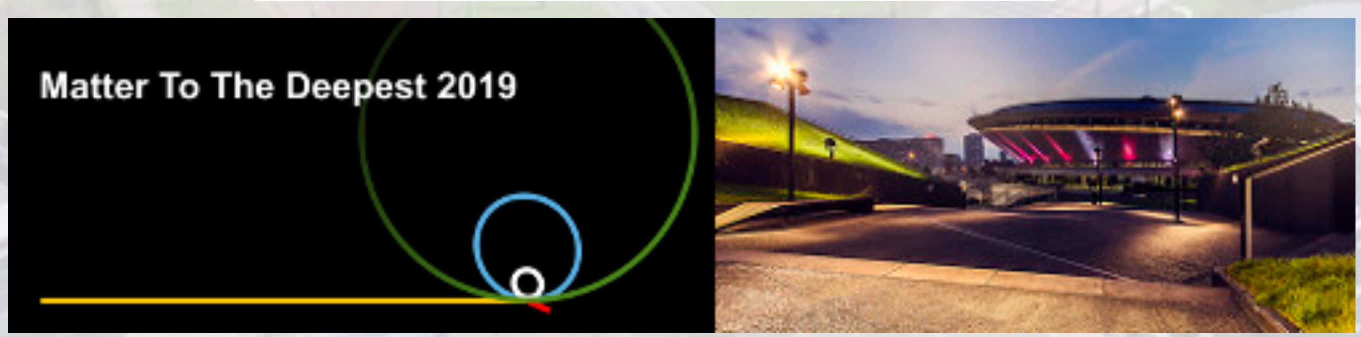


Katowice, 2019 September 4.

Introduction

Bump hunt at LHC was successful: we found the Higgs boson.

On the downside: no other sharp resonance $\Longrightarrow$ the zoo of particle physics does not want to expand

The need for high precision predictions is higher than ever!

In the absence of sharp peaks we have to rely on precision predictions to tell minute differences from our models

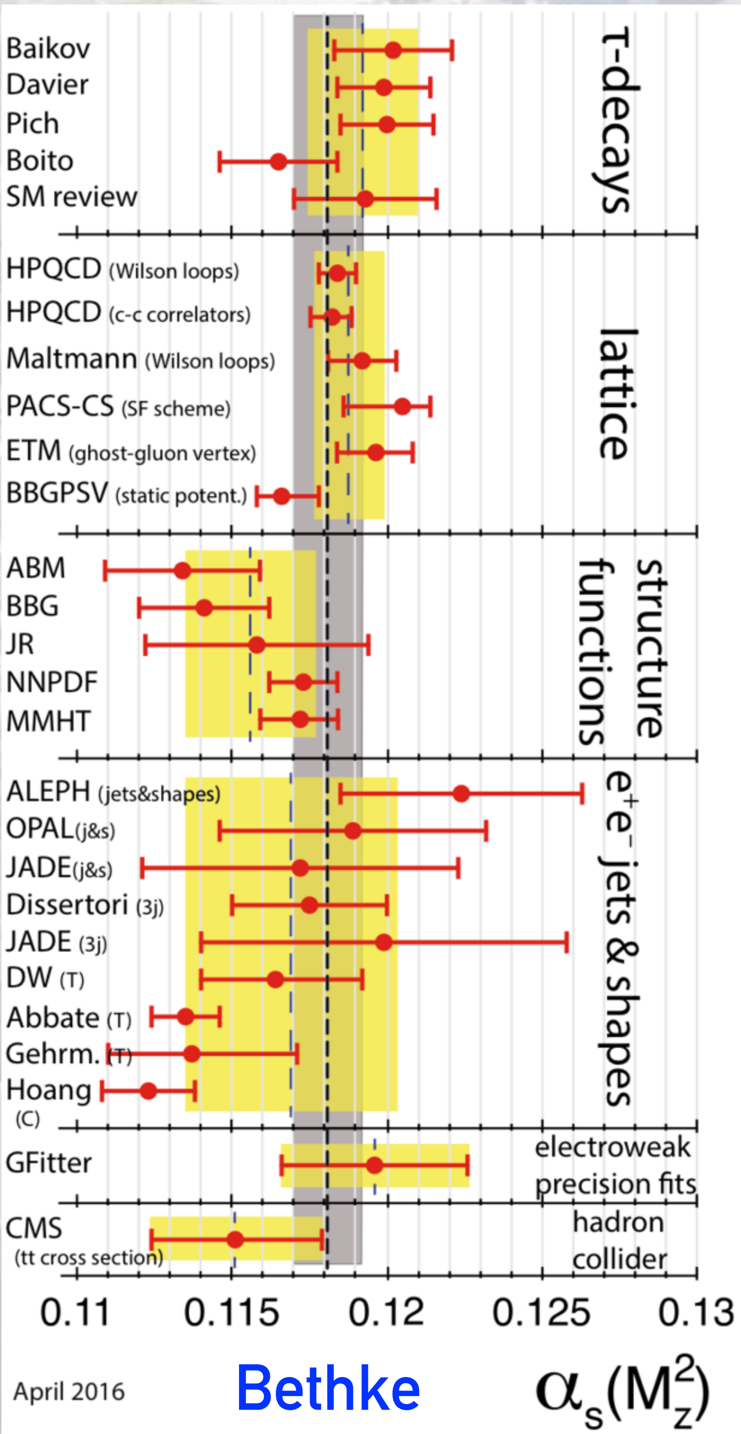
Example: Indirect top mass determination using Peskin-Takeuchi parameters: $m_t \in [145, 185]$ GeV.

$\Longrightarrow$ Not just the measurement has to be precise but predictions as well for both signal and background processes!

Introduction

Not just discovery requires precision:

Our models can have several free parameters: masses, couplings, PDFs, etc.



These have to be determined with the greatest available accuracy

One such parameter is the strong coupling of QCD

Being part of the Standard Model the most precise determination of strong coupling is fundamental and of paramount importance

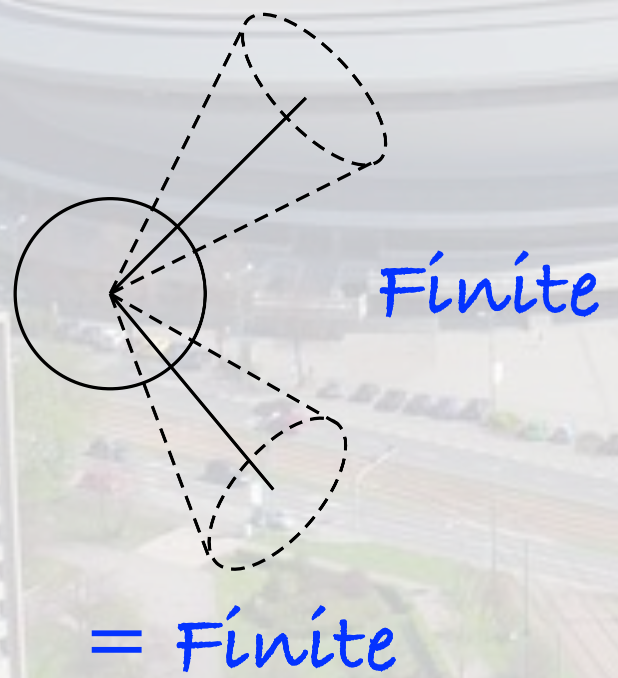
$\Longrightarrow$multi-fold interest for precision calculations

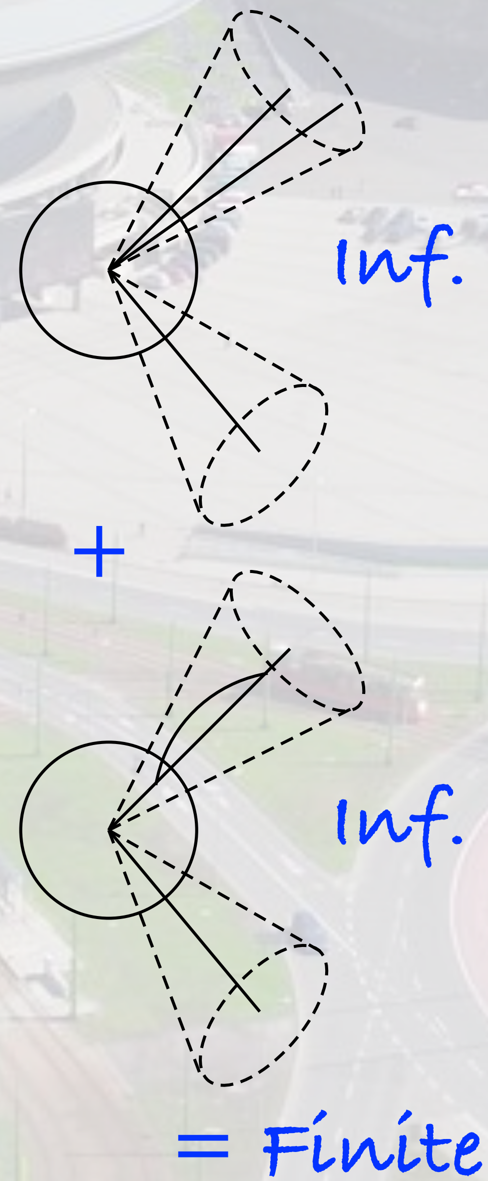
A question remains: what is actually precision?

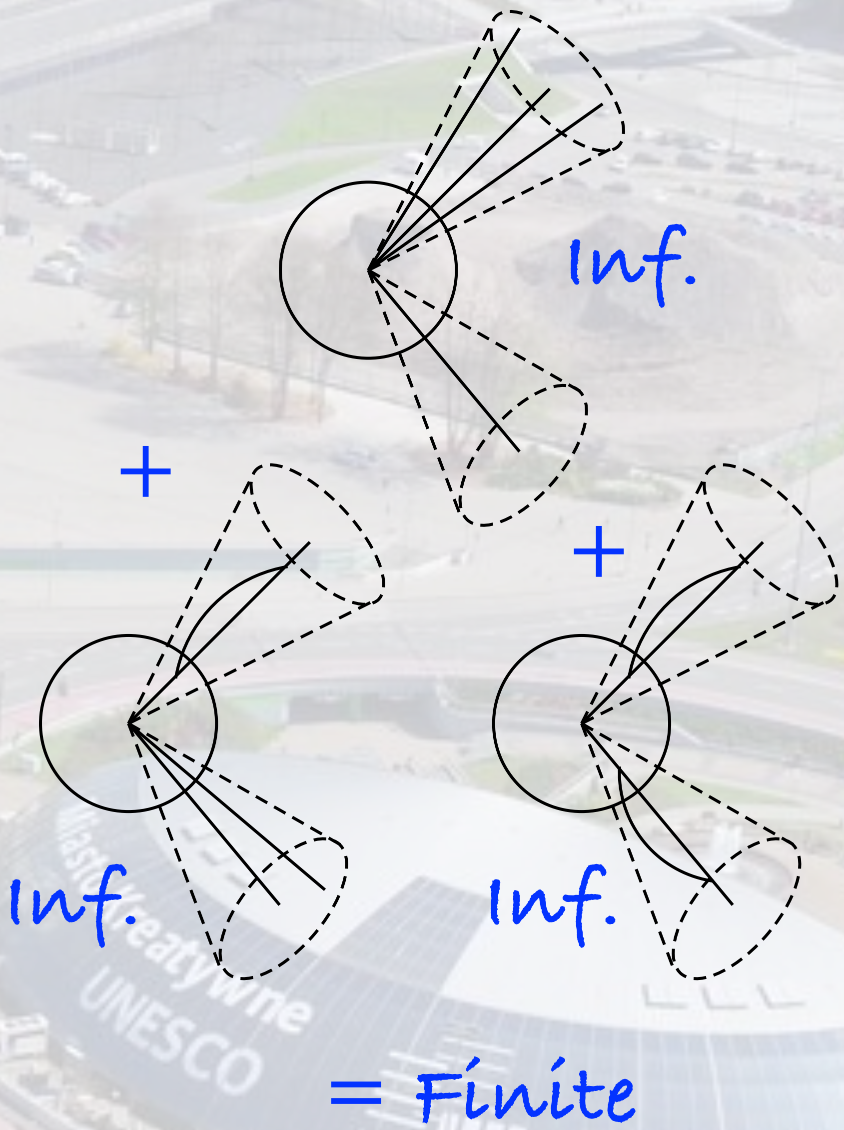
Introduction

The de jure method for calculations in high energy particle physics is perturbation theory:

$$O[F] = \left(\frac{\alpha_S}{2\pi}\right)^n A[F] \quad + \quad \left(\frac{\alpha_S}{2\pi}\right)^{n+1} B[F] \quad + \quad \left(\frac{\alpha_S}{2\pi}\right)^{n+2} C[F] + \cdots$$







Order-by-order the number of real and/or virtual emissions increase

Extra radiation introduce kinematic singularities

Introduction

For IR safe observables only the sum of all contributions is finite order-by-order!

The problem is the complexity: phase space integrals can only be done numerically!

Consider a simple example:

$$I = \lim_{\epsilon\to 0}\left(\int_0^1 \frac{\mathrm{d}x}{x^{1-\epsilon}}F(x) - \frac{1}{x^{\epsilon}}F(0)\right)$$

Two traditional ways exist to deal with the situation:

Slicing:

$$I \sim \lim_{\epsilon\to 0}\left(F(0)\int_0^{\delta}\frac{\mathrm{d}x}{x^{1-\epsilon}} + \int_{\delta}^1 \frac{\mathrm{d}x}{x^{1-\epsilon}}F(x) - \frac{1}{\epsilon}F(0)\right) = F(0)\log(\delta) + \int_{\delta}^1 \frac{\mathrm{d}x}{x}F(x)$$

Subtraction:

$$I = \lim_{\epsilon\to 0}\left(\int_0^1 \frac{\mathrm{d}x}{x^{1-\epsilon}}\left[F(x) - F(0)\right] + F(0)\int_0^1 \frac{\mathrm{d}x}{x^{1-\epsilon}} - \frac{1}{\epsilon}F(0)\right) = \int_0^1 \frac{\mathrm{d}x}{x}\left[F(x) - F(0)\right]$$

Introduction

To assess second order corrections in QCD we use the CoLoRFuLNNLO scheme (Del Duca, Somogyi & Trócsányi) which is a subtraction scheme

Subtractions are formulated by applying soft and/or collinear factorization properties of QCD amplitudes

These are equipped with momentum mappings from multi-emission to Born kinematics and various momentum fractions

Bear in mind that this is a solution to the problem! Other schemes are available as well:

- Antenna
- STRIPPER
- Projection to Born
- Jettiness slicing
- Loop-tree duality
- …

QCD is Alive and Kicking

QCD is important for multiple reasons:

QCD processes can be irreducible backgrounds to several interesting processes, like for t t~ H production in the H$\rightarrow$ b b~ channel (t t~ b b~ production)

LHC is a hadron-hadron machine…we have underlying events, soft content, heavy hadronic activity, jets are made of hadrons from partons through hadronization

Gauge field of QCD is non-abelian $\Longrightarrow$ calculations are beautifully complex

Computations are worth performing for their sheer beauty

Not to mention that we are in an era where these extremely complex computations can be handled both numerically and analytically

QCD is Alive and Kicking

In certain areas our understanding is limited: like in hadronization

So far non-perturbative corrections cannot be assessed from first principles, only phenomenological models exist

A dispersive model exists (Dokshitzer, Marchesini & Webber) with a recent facelift with an alternate effective coupling (Catani, De Florian & Grazzini)

Model uncertainties of power corrections propagate to final results hampering overall uncertainties in precision measurements

Two options remain:

- better understand power corr's: derivation from first principles
- getting rid of them: choosing observables which are not sensitive to them

Observables of Soft-Drop Variety

Soft drop offers a way to get rid (partially) of power corrections:

1) Perform a jet clustering according to one of the algorithms keeping full track of pseudojet mergings

2) Consider the last merging and apply condition of:

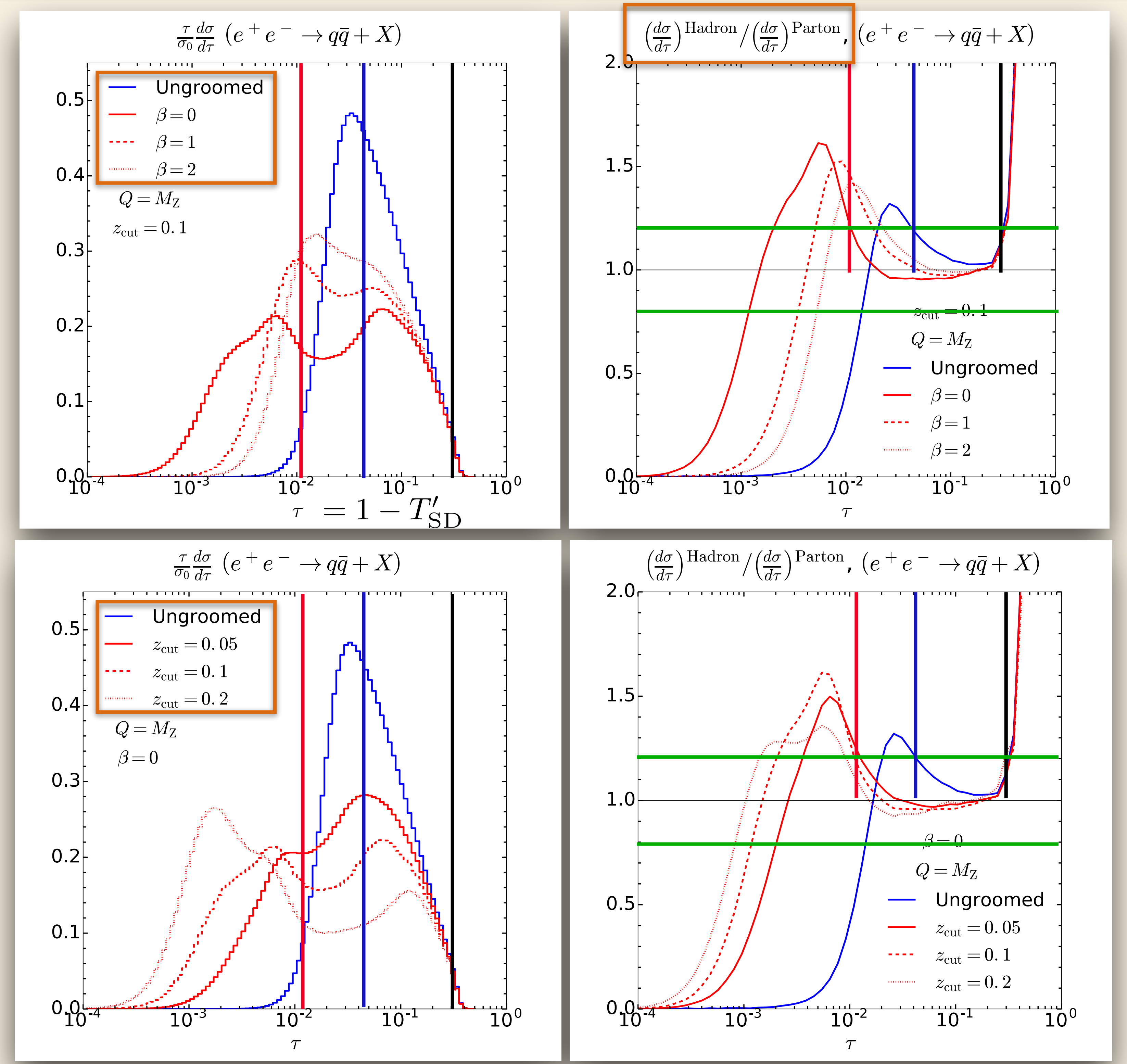
$$\frac{\min\left[E_i, E_j\right]}{E_i + E_j} > z_{\rm cut}\left(1-\cos\theta_{ij}\right)^{\beta/2} \quad \text{or} \quad z_{\rm cut}\left(\frac{1-\cos\theta_{ij}}{1-\cos R}\right)^{\beta/2}$$

3) If condition fails drop the softer pseudojet and consider the next merging in the remaining one, if passes apply condition to both merges

4) Recursively apply the condition until reaching initial tracks

5) In any further analysis only consider those tracks which survive the soft-drop condition by keeping intact those pseudojets which they are building up

Soft drop thrust had. corr. by PYTHIA



Zoltán's slide from EPSHEP2019[17]

J. Baron et al, arXiv: 1803.04719

Observables of Soft-Drop Variety

For soft-drop observables resummed results can also be obtained

But it is always nice to be able to check the result...

A possible check:

The exponentiated result is expanded and log structure is checked with high-precision computations in region where resummation is important (very small or very close values to one)

Rule of thumb:

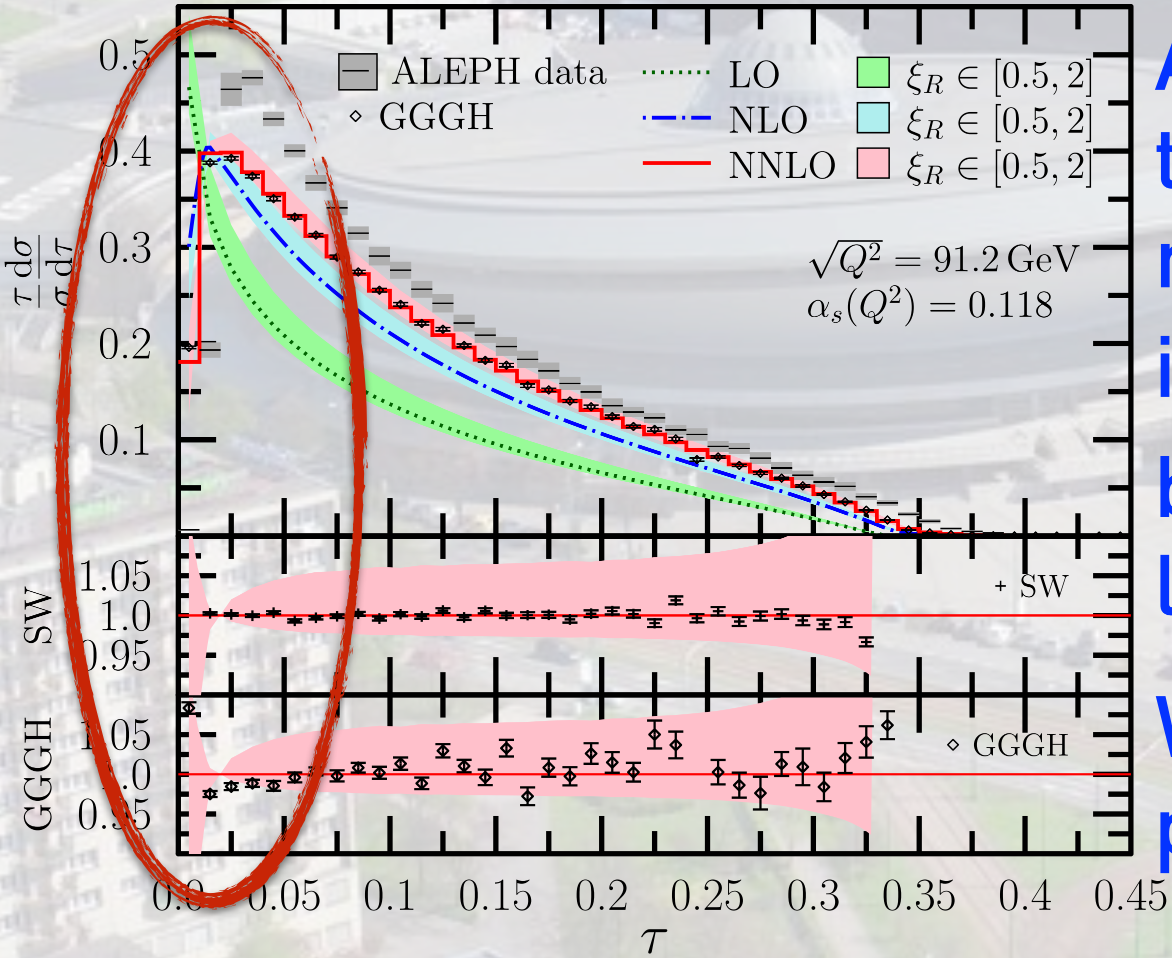
If the resummation is carried out to $N^{k+1}LL$ we need an N^kLO computation to check logs

Nowadays it became very common to have k=2 $\Longrightarrow$ large demand for precise, numerically stable NNLO computations

The computation has to be numerically stable even for values close to kinematic limit (where resummation is understood) to really see the logs

Observables of Soft-Drop Variety

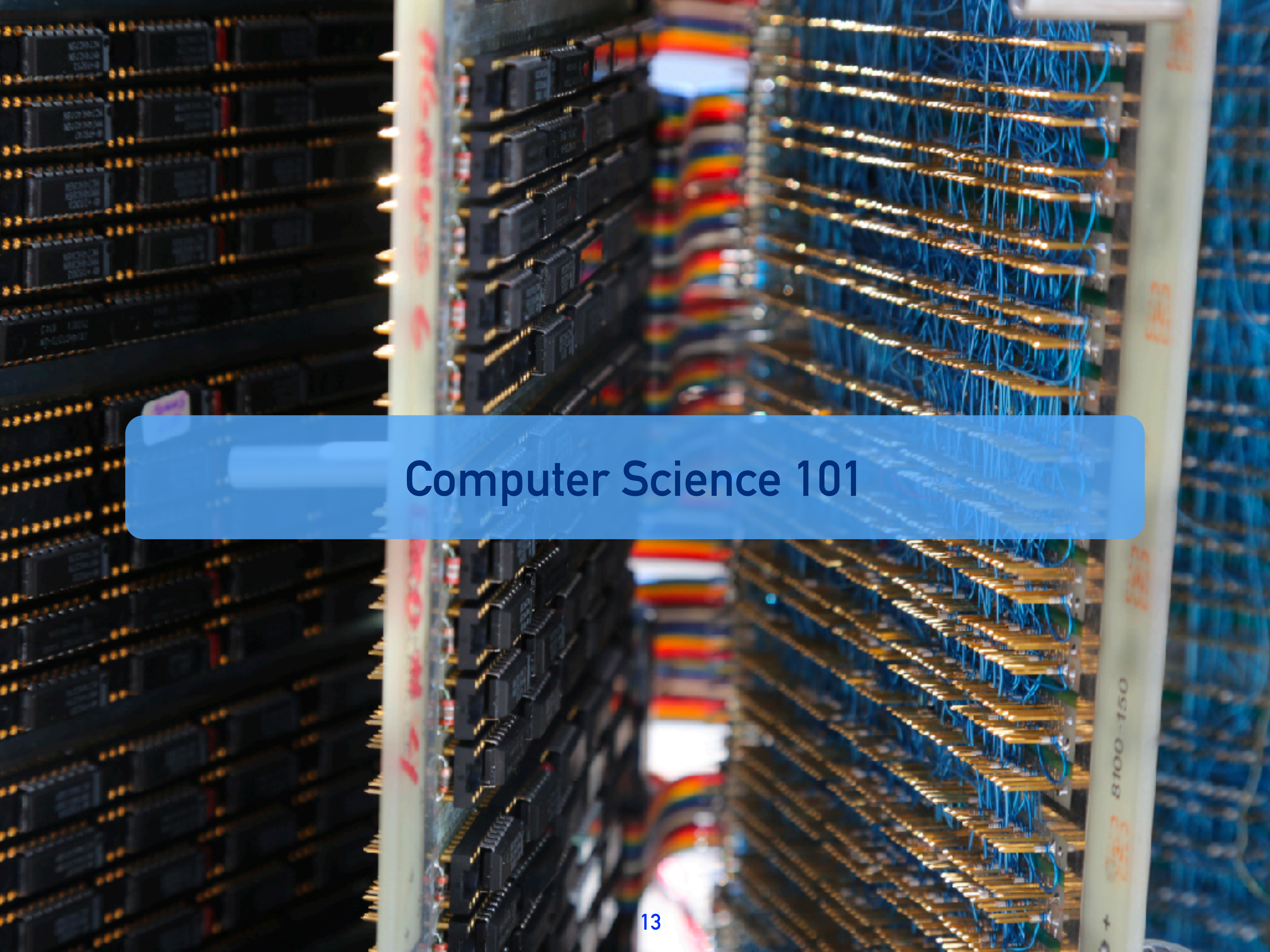
Note: in the NNLO computation we have to go very-very close to kinematic limit



As going closer and closer to the limit the kinematic singularities become more and more exposed. Computation is done with subtractions $\Longrightarrow$ Contributions become larger and larger with larger and larger subtractions

We have to have the difference calculated precisely!

Why should we worry that much?



Computer Science 101

Computer Science 101

In modern computers floating point arithmetics is implemented using the IEEE754 standard, stored in 1+m+l bits:

$$\underbrace{\square}_{\text{sign}}\ \underbrace{\square\ldots\square}_{\text{exponent}}\ \underbrace{\square\square\ldots\square}_{\text{fraction}}$$

An n-bit floating point number consists of 1 bit (sign) + m bit (exponent) + l bit (fraction)

Exponent: in terms of powers of two: $\in \left\{2^{2^{m-1}-1}, \ldots, 2^{2^{m-1}}\right\}$

Fraction: in terms of powers of two: $\in \{1, 1+2^{-l}, \ldots, 2-2^{-l}\}$

$\Longrightarrow$We have a fixed number of digits

$\Longrightarrow$Single precision: 8 digits

$\Longrightarrow$Double precision: 16 digits

Computer Science 101

A simple example:

$$\begin{array}{r} 1.254657 \cdot 10^{-5} \\ +2.346789 \cdot 10^{-9} \\ \hline ??? \end{array}$$

We have to align:

$$\begin{array}{r} 1.254657 \cdot 10^{-5} \\ +0.0002346 \cdot 10^{-5} \\ \hline 1.257003 \cdot 10^{-5} \end{array}$$

Alignment forces you to loose precision!

We lost precision due to difference in magnitude!

The same happens when a subtraction scheme is used: after subtraction the resulting difference is orders of magnitude smaller than the original contributions!

Does Your Number Make Sense?

Close to kinematic limit the SME can become enormously large, so does the subtraction terms!

⟹Contribution after subtraction can only have a couple of meaningful digits!

We can also loose accuracy when the SME is calculated!

⟹ can partially ruin cancellation!

⟹can result in with bins with large content with large uncertainty!

⟹This must be avoided!

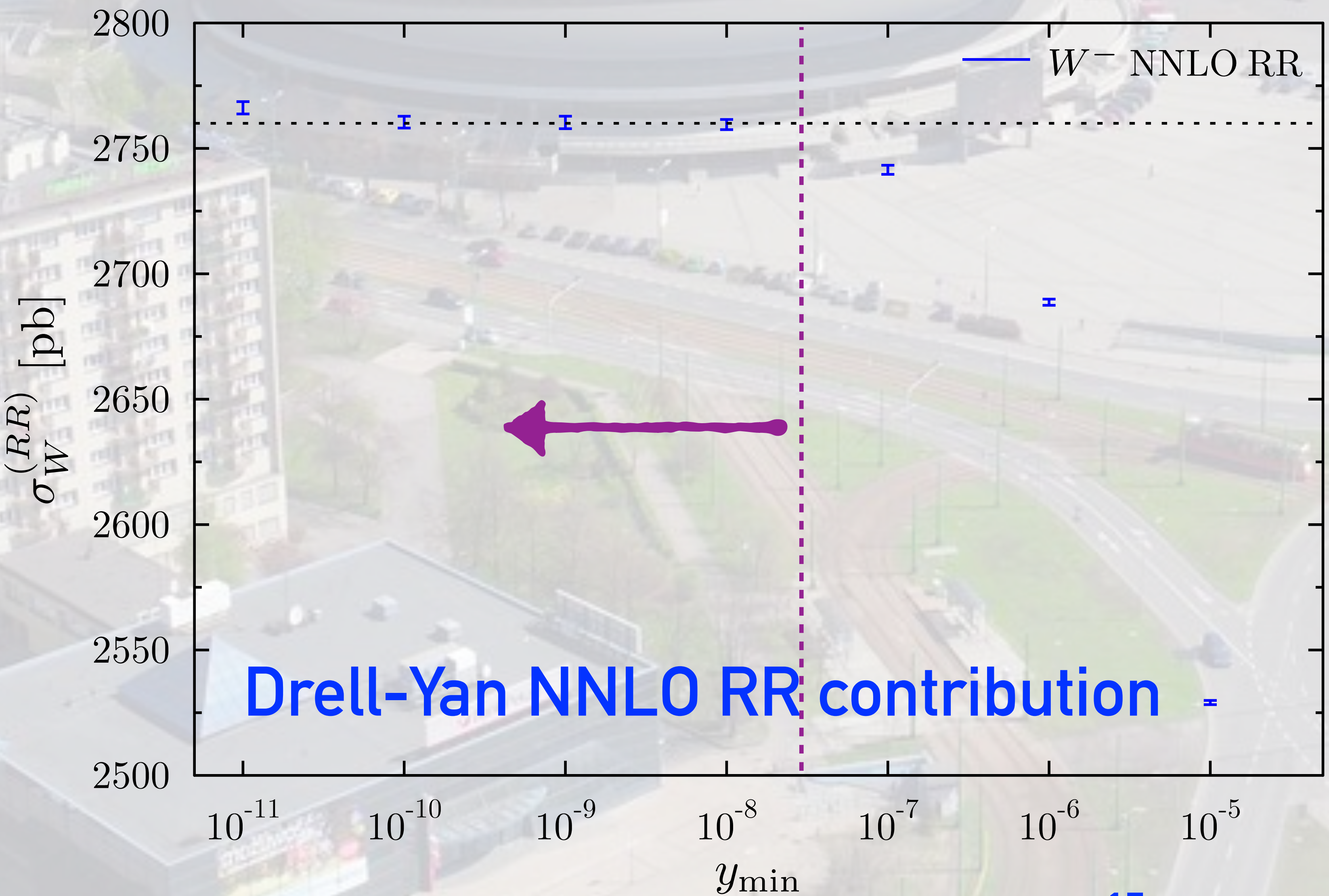
Possible solution: avoid the edges of phase space!

Does Your Number Make Sense?

Possible solution: avoid the edges of phase space!

Two-particle invariants can be limited from below:

$$\min_{i,j} \frac{p_i \cdot p_j}{p_\oplus \cdot p_\ominus} > y_{\min}$$



Important Note: $y_{\min}$ should be chosen that its variation leaves the cross section intact!

This gives an optimum in meaningfulness and numerical stability

$y_{\min}$ cut is usually dubbed as technical cut

Does Your Number Make Sense?

Examples:

- EVENT2: CUTOFF
- POWHEG-BOX: par_isrtiny* * *, par_fsrtiny* * *
- Your favorite beyond LO code…

If an NNLO calculation is used to check log structure coming from resummation a priori we cannot tell which technical cut value allows for a fair comparison!

As an example we can consider the soft-dropped version of the heavy hemisphere jet mass

Soft-Dropped Heavy Hemisphere Mass

Sticking to β=0, original jet clustering to find hemispheres is according to k_T algorithm

In both hemispheres Cambridge/Aachen algorithm is run to find merging history

Soft drop criterium is applied to pseudojets:

$$\frac{\min\left[E_i, E_j\right]}{E_i + E_j} > z_{\mathrm{cut}}$$

Remaining tracks are used to calculate the hemisphere mass:

$$\rho = \frac{\max\left[m_R^2, m_L^2\right]}{E_J^2}$$

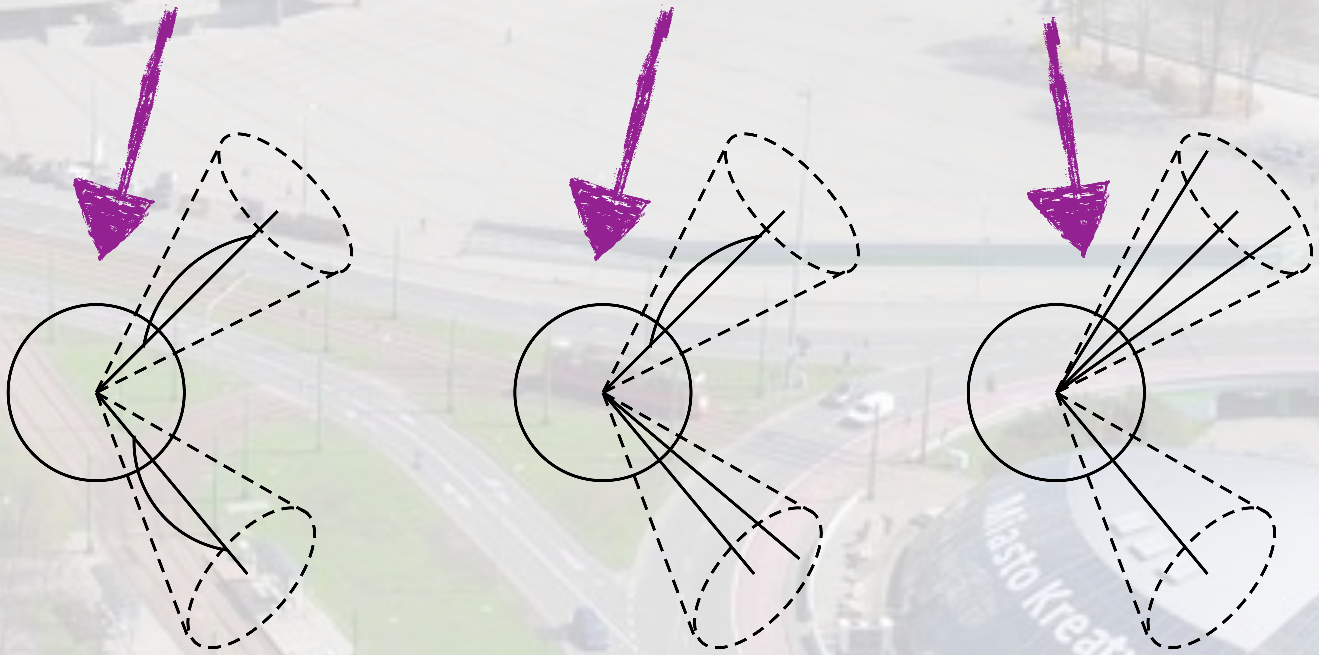
Soft-Dropped Heavy Hemisphere Mass

The perturbative expansion of soft-dropped heavy hemisphere mass in terms of α_S:

$$\sigma[\rho] = \sigma^{\mathrm{LO}}[\rho] + \sigma^{\mathrm{NLO}}[\rho] + \sigma^{\mathrm{NNLO}}[\rho] + \dots$$

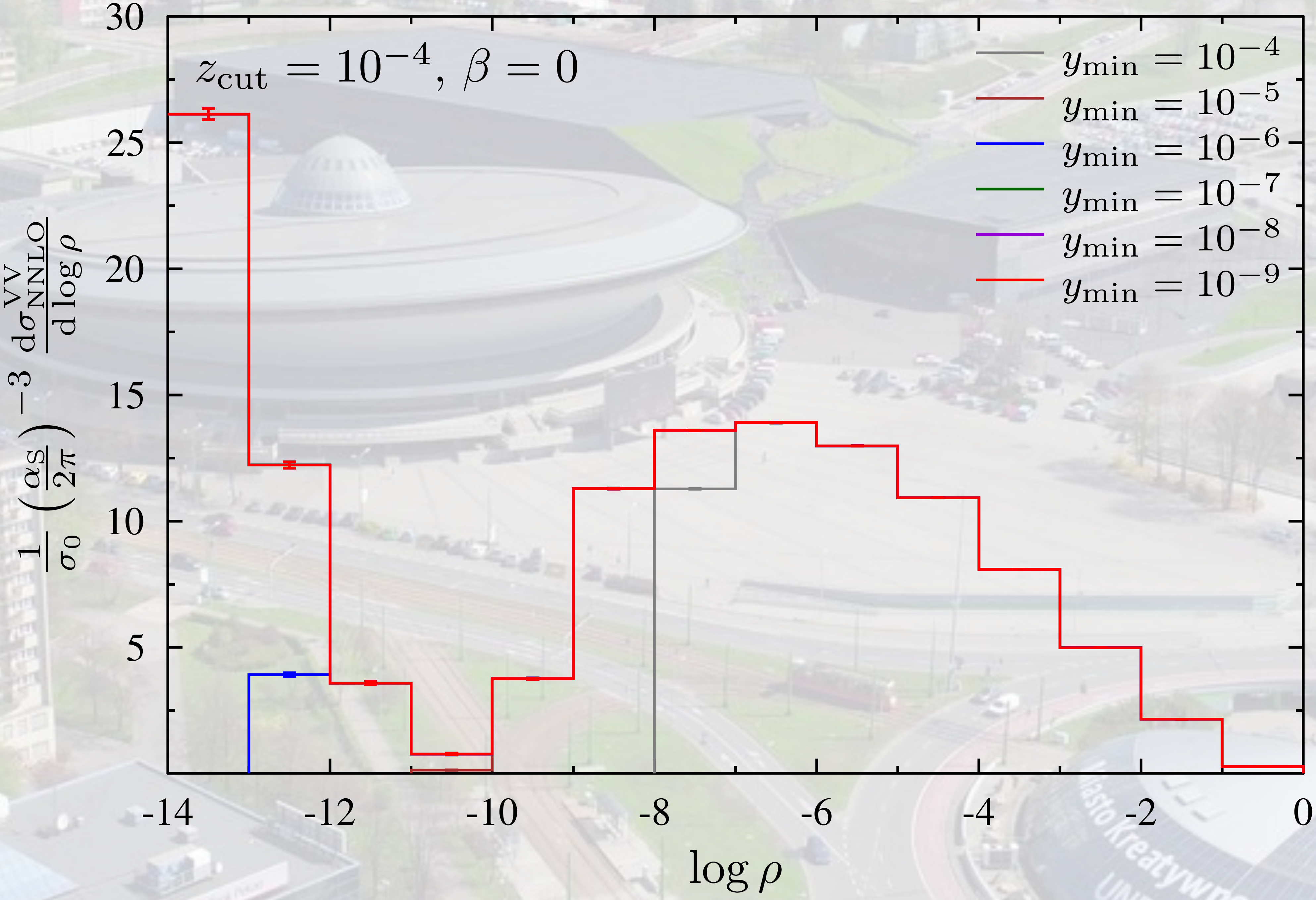
The NNLO contribution can be further dissected:

$$\sigma^{\mathrm{NNLO}}[\rho] = \sigma^{\mathrm{VV}}_{\mathrm{NNLO}}[\rho] + \sigma^{\mathrm{RV}}_{\mathrm{NNLO}}[\rho] + \sigma^{\mathrm{RR}}_{\mathrm{NNLO}}[\rho]$$



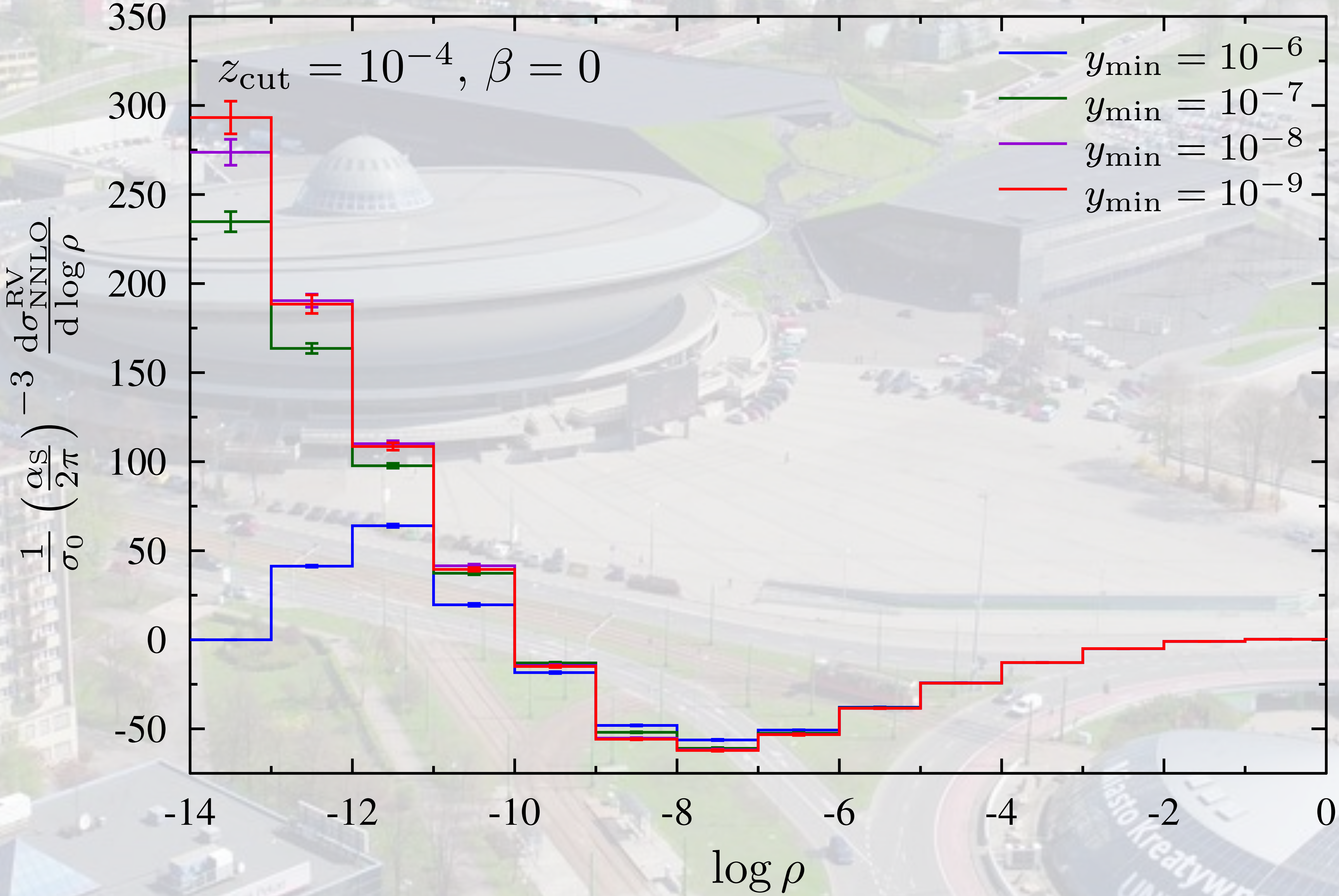
As number of unresolved emissions increase the need for a technical cut becomes more and more severe

Soft-Dropped Heavy Hemisphere Mass



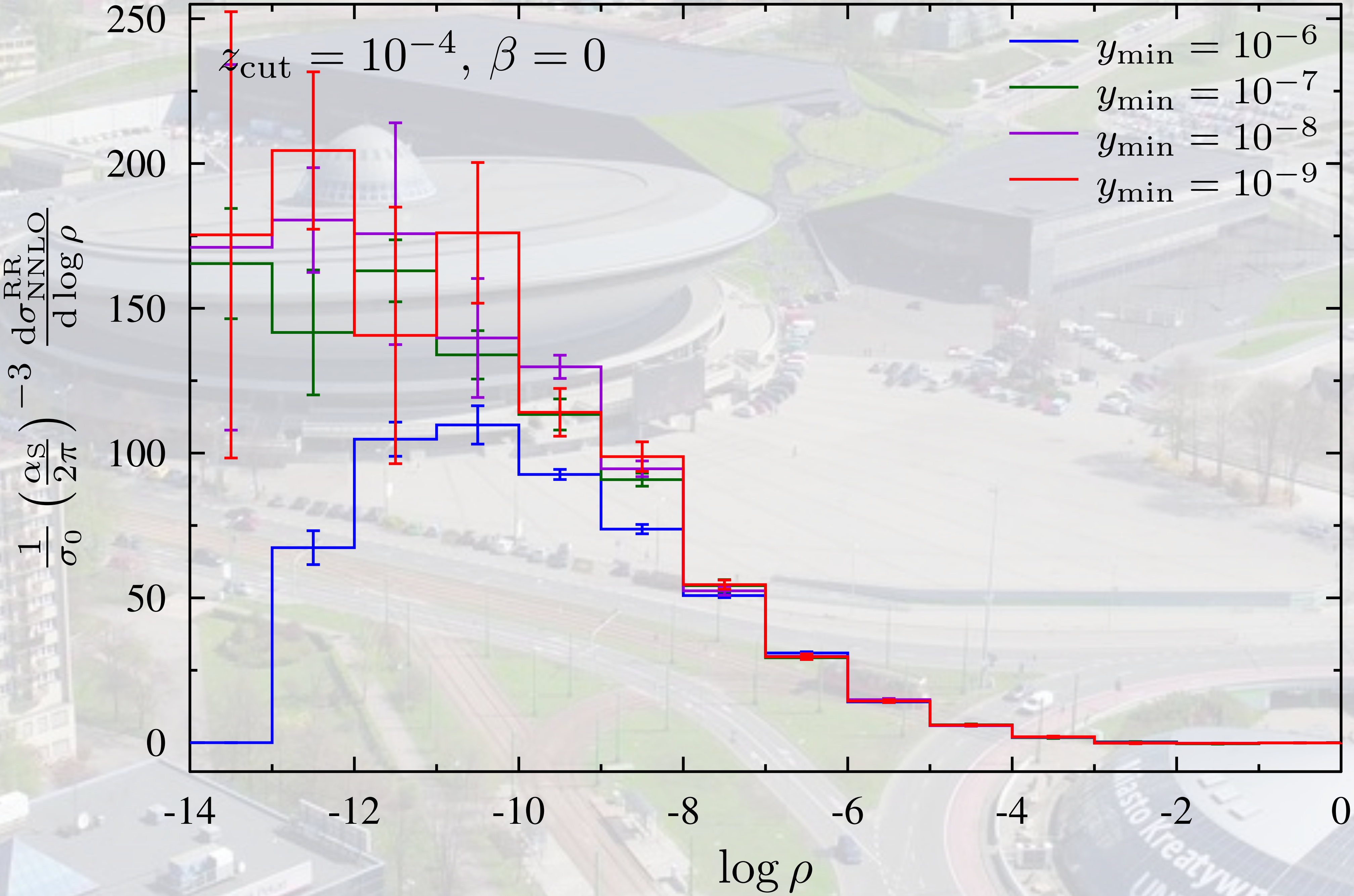
VV contribution at different technical cut values

Soft-Dropped Heavy Hemisphere Mass



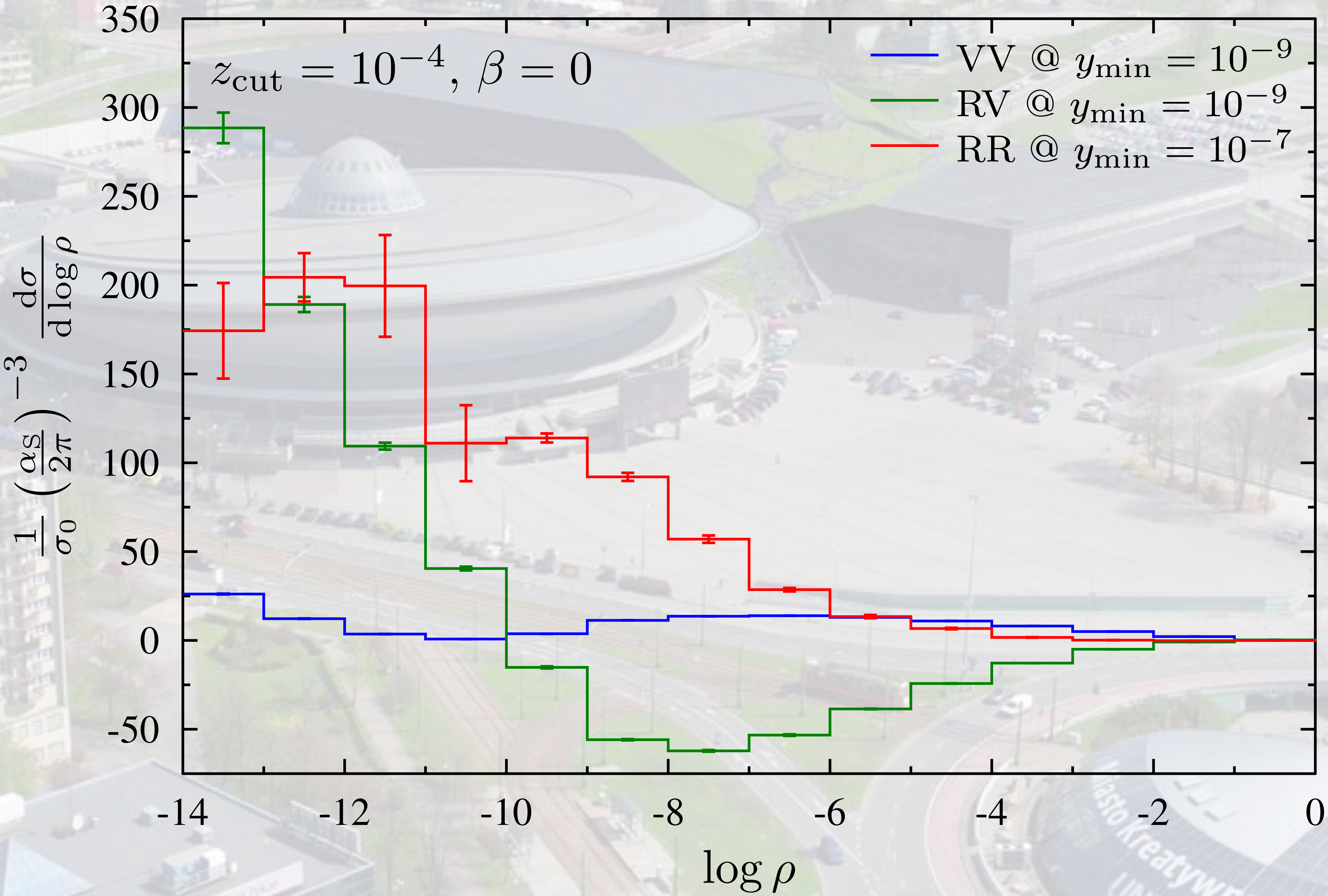
RV contribution at various technical cut values

Soft-Dropped Heavy Hemisphere Mass



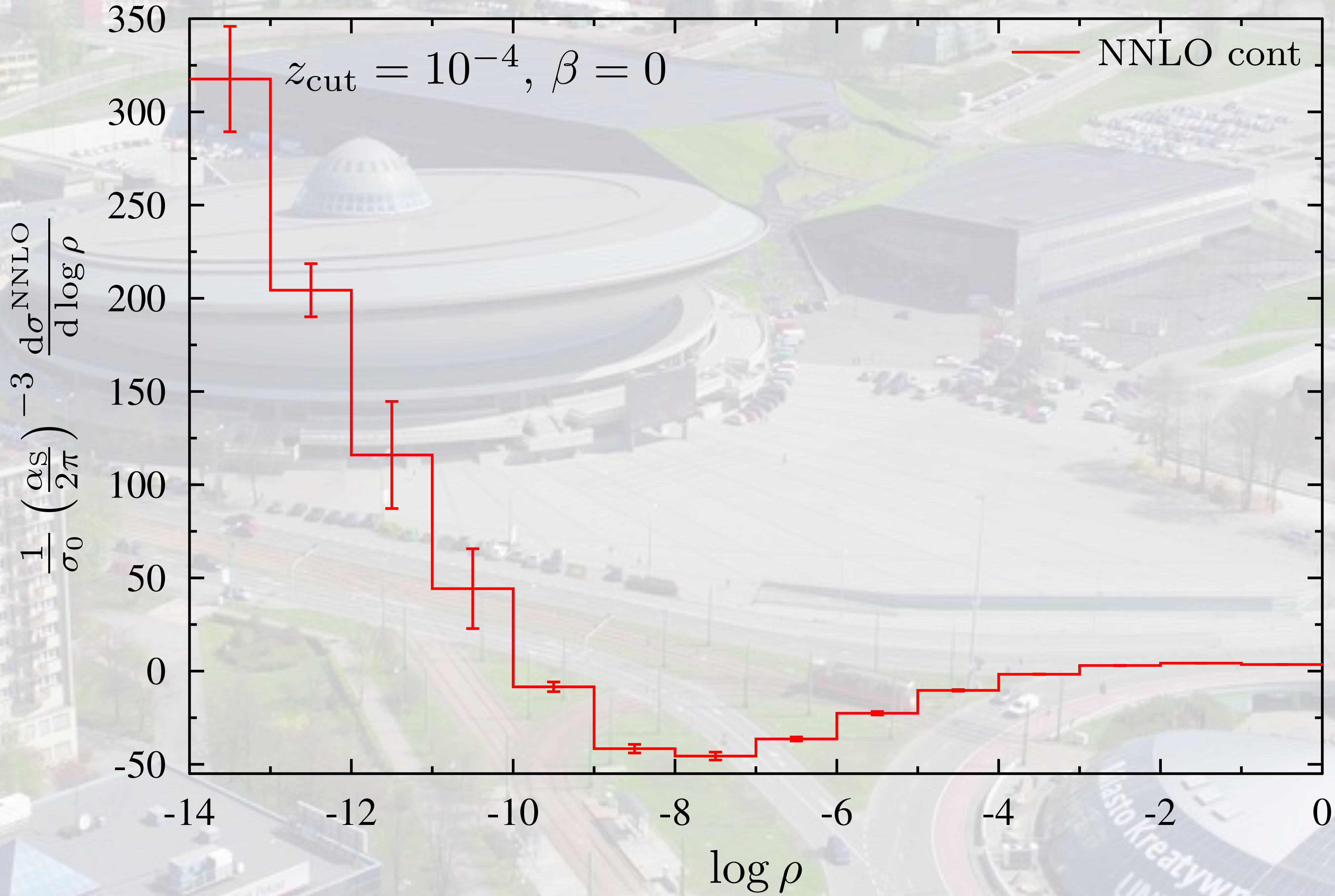
RR contribution at various technical cut values

Soft-Dropped Heavy Hemisphere Mass



The three NNLO contributions obtained at their best $y_{\min}$ values

Soft-Dropped Heavy Hemisphere Mass



The total NNLO contribution to the soft-dropped heavy hemisphere mass

Conclusions

- Technical cuts are frequently used in beyond LO computations but seldomly mentioned
- Any number coming out of a beyond LO computation should treated with scrutiny
- Can only be sure of the result if it does not show any dependence on the used technical cut
- In case of slicing the situation is more elaborate: not just the slicing parameter should be selected to be a small value but the technical cut as well $\Longrightarrow$ saturation should be shown on a 2D domain
- It is possible to calculate observables @ NNLO for small values but extra work has to be done to be sure about the physicality of obtained numbers

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