FORCING, EXOTIC SMOOTHNESS, AND PHYSICS

Torsten Asselmeyer-Maluga[2] • Krzysztof Bielas[1] • Paweł Klimasara[1] • Jerzy Król[1]

[1]Institute of Physics, University of Silesia in Katowice, Uniwersytecka 4, 40-007 Katowice, Poland
[2]German Aerospace Center , Rutherfordstraße 2, 12489 Berlin, Germany

torsten.asselmeyer-maluga@dlr.de krzysiek.bielas@gmail.com p.klimasara@gmail.com iriking@wp.pl

Mathematical Preliminaries

ZF(C)=Zermelo-Fraenkel set theory (with the axiom of Choice)
It is widely accepted paradigm that most of the mathematics required by theories of physics is based on formal theory of sets. ZFC has infinitely many (if there exists at least one!) non-isomorphic models of various cardinalities (so there is no unique universe for mathematics!). Especially important, also for our considerations, is the class of models called Countable Transitive Models (CTMs).

Model of a theory is a mathematical universe (some class/set) within which the theorems of this theory are fulfilled.

Real numbers in CTMs of ZFC

Some real number from one CTM need not to be a real number in another CTM. That is, (for *some* models) one has: Different models of ZFC ⇔ Different sets of reals.

$\mathbb{R}$ - the set of reals (in metatheory), it contains real numbers from *every* CTM of ZFC
$\mathbb{R}_{\mathcal{M}}$ - the set of reals in some specified CTM denoted $\mathcal{M}$

So for any CTM $\mathcal{M}$ of ZFC there is always $\mathbb{R}_{\mathcal{M}} \subset \mathbb{R}$.

A set-theoretic *forcing* is a formal technique that can be seen as a tool to "switch" between the models. The specific forcing depends on the choice of an appropriate Boolean Algebra (BA) and *generic ultrafilter G*. Starting from some CTM $\mathcal{M}$ and $G \notin \mathcal{M}$ forcing gives rise to a CTM $\mathcal{M}[G]$ called its generic extension for which $G \in \mathcal{M}[G]$. In particular "new" reals are added. For Cohen forcing $\mathbb{R}_{\mathcal{M}} \subset \mathbb{R}_{\mathcal{M}[G]}$ and the set $\mathbb{R}_{\mathcal{M}}$ is dense in $\mathbb{R}_{\mathcal{M}[G]}$ (and has Lebesgue measure 0!). Similarly $\mathbb{R}_{\mathcal{M}}$ is dense and of measure 0 in the full real line $\mathbb{R}$.

Motivations

- Common use of real numbers in physics (actual results of measurements, spacetime parametrization...)
- Quantum Mechanics (QM) ↔ General Relativity (GR) Incompatibility
- No signs of supersymmetry nor extra dimensions in up-to-date accelerator experiments

The Latent Meaning of Forcing in QM

All undermentioned results relate to *Boolean contexts*. That is, using the spectral theorem, we switch the algebra of usual self-adjoint operators on Hilbert space into the algebra of operators that are a multiplication by a real measurable function. Obtained BA is an *atomless measure algebra* and hence it relates to forcing. This forcing is crucial for QM and GR relation!

Since QM cannot be sufficiently formalized within a single model of ZFC [1], we try the description through a varying model (the original idea of *dynamical network of models* by J. Król). Quantum measurement, having the reals as outcome, is related to a change of a model via so-called *random* forcing.

A brief history of forcing in QM

Paul A. Benioff (1976)
There is no single model of ZFC that can be used for the correct representation of mathematics of QM along with its statistical predictions [1].

Robert A. van Wesep (2006)
If there exists the semiclassical state realizing the *Local Hidden Varibles* program, then it is a generic ultrafilter on appropriate BA made of projections from the usual *lattice of projections* [2].

Although Wesep's semiclassical state is not allowed in full QM, in the case of *commuting observables* the LHV parameters do exist. One reason is the fact that Bell's states can not be constructed in the commuting case and Bell's inequalities must be fulfilled.

Micro to macroscale shift: J. Król, P. Klimasara (2015)

Suppose that the real numbers which parametrize space come from the quantum realm via continuous measurement (~ *position observable*). Then there exists a nontrivial forcing on the measure algebra on $\mathbb{R}^3$ grasping the difference between reals used in QM and the continuum of real parameters describing the spacetime in the large scales (GR).

*See details in [3].

The dual perspective

A few crucial definitions and important facts:

$\mathsf{N} = \{x \subset \mathbb{R} \mid \mu(x) = 0\}$, where μ is usual Lebesgue measure
$\mathsf{M} = \{x \subset \mathbb{R} \mid x \text{ is meager}\}$

Here a *meager* set is a subset of $\mathbb{R}$ that is in precise sense of topologically neglible size, namely it is a countable union of nowhere dense sets (ones that are *not* dense in every neighbourhood).

$\mathfrak{Bor}(\mathbb{R})$ - Borel subsets of $\mathbb{R}$

The Sierpiński-Erdős duality theorem

Under CH* there exists a bijection satisfying:

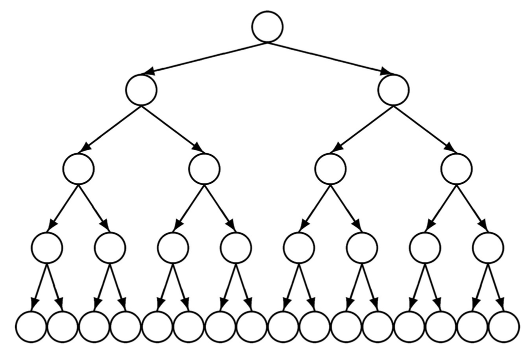
$$f : \mathbb{R} \xrightarrow{1-1} \mathbb{R}$$
$$f = f^{-1}$$
$$f(x) \in \mathsf{N} \Leftrightarrow x \in \mathsf{M}$$
$$f(x) \in \mathsf{M} \Leftrightarrow x \in \mathsf{N}$$

Measure algebra $\mathfrak{Bor}(\mathbb{R})/\mathsf{N}$ is related to random forcing.

Cohen algebra $\mathfrak{Bor}(\mathbb{R})/\mathsf{M}$ is related to the so-called Cohen forcing.

Using above duality principle, we can switch measure algebra, obtained from QM, to the Cohen algebra. Both are Boolean algebras underlying the forcing constructions. Even though measure algebra is not absolute (that is, it depends on the choice of a model of ZFC), Cohen algebra is absolute in a sense of being unique (up to isomorphism) atomless complete Boolean algebra with a *countable dense subset*. Measure algebra has no such subset.

The full binary tree:



etc...

It represents Cantor set $2^\omega \simeq \mathbb{R}$ and Casson handles. Nodes correspond to the countable dense subset.

Such infinite constructions relate to geometric objects called *Casson Handles* that are specific to *exotic* smooth 4-manifolds (like $\mathbb{R}^4$).

We can use exotic smooth $\mathbb{R}^4$ (instead of the standard one as usually in GR) while building cosmological models. There was proposed they can resolve some basic comological problems like dark matter or dark energy [7]. Moreover, as shown by T. Asselmeyer-Maluga and J. Król [5], taking exotic smooth manifold as spacetime model one can obtain realistic values (as compared with recent Planck data and effective models like Starobinski's or R^2):

- the CC value,
- the shape of the primordial inflation potential,
- the expansion rate of the Universe.

An exotic $\mathbb{R}^n$ is a differentiable manifold that is homeomorphic but not diffeomorphic to the Euclidean space $\mathbb{R}^n$. Interesting fact is that for any positive integer $n \neq 4$, there are no exotic smooth structures on $\mathbb{R}^n$! The spacetime dimension is special. Moreover, there exists a *continuum* of such structures.

*Continuum Hypothesis: $|\mathbb{R}| = 2^{\aleph_0} = \aleph_1$.

Summary

Our originally disjoint research on forcing in quantum mechanics and exotic 4-manifolds in cosmology meet unexpectedly on a formal ground, supported by a well known in mathematics measure and category correspondence (The Sierpiński-Erdős duality theorem). While, connected with micro to macroscale shift, random forcing solves in a tricky way the particle physics overestimation problem for zero modes of quantum fields, the dual picture of exotic geometries completes the idea by giving the nonzero and realistic CC value. The consistency of a whole model still needs many issues to check (like whether chosen exotic manifold should be *large* or *small*), but the first results and many interesting mathematical problems arising from it supports our belief that it is worth further exploring.

Plans for Future Work

- Determining the exact relation of deformed binary trees obtained from QM and appropriate exotic manifold given by them via connected Casson handles.
- Attempt to find the impact of meager real line on the various distortions of CMB.

The Cosmological Constant Problem

The discrepancy of CC prediction in particle physics with observations comes from vastly overestimated contributions (to CC) from zero-modes of quantum fields. In our approach, relating early spacetime evolution with the change of CTMs of ZFC, such contributions completely vanish [4].

The zero-point energy of quantum field corresponding to a particle of mass m:

$$\frac{E}{V} = \int_{\mathbb{R}^3_{\mathcal{M}}} \frac{d^3k}{(2\pi)^3} \frac{\sqrt{k^2+m^2}}{2}, \quad m \in \mathbb{R}_{\mathcal{M}[G]}, \; \mathrm{k} \in \mathbb{R}^3_{\mathcal{M}}.$$

Here all such integrals equal 0 since we integrate over the null set $\mathbb{R}^3_{\mathcal{M}} \subset \mathbb{R}^3_{\mathcal{M}[G]} \subset \mathbb{R}^3$.

Nevertheless, the CC cannot be zero to fit the experimental data. To obtain nonzero and realistic value of CC one can turn to exotic geometries.

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