Radius Stabilization and Dark Matter with a bulk Higgs in warped extra dimension

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Outline

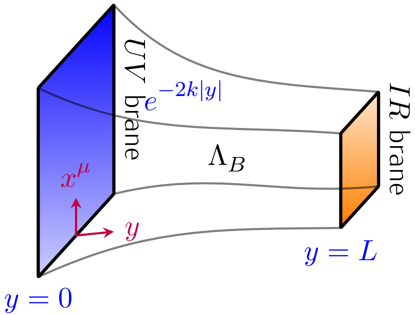
- Randall-Sundrum model: a brief review.
- Radius stabilization with a bulk Higgs doublet in 5D warped models
- $\mathbb{Z}_2$ symmetric warped extra-dimension: the IR–UV–IR model.
- Warped Higgs dark-matter.
- Summary.

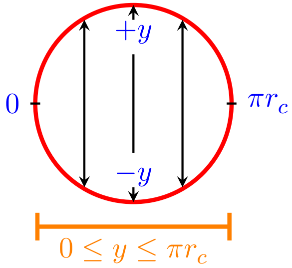
★ AA, B. Grzadkowski, J.F. Gunion and Y. Jiang, "Higgs Dark Matter from a Warped Extra-Dimension – *the truncated-inert-doublet model*", arXiv:1504.03706, *to appear in JHEP*.

★ AA, B. Dillon and B. Grzadkowski, "Higgs-Radion Unification and Electroweak Precision Observables in Warped Extra Dimension", in progress.

Randall–Sundrum model: a brief review

RS proposed a 5D model with two D3-branes on $S_1/\mathbb{Z}_2$ orbifold along the extra-dimension to solve hierarchy problem. hep-ph/9905221





$$ds^2 = e^{-2k|y|}\eta_{\mu\nu}dx^\mu dx^\nu + dy^2$$

$$k \equiv \sqrt{-\Lambda_B/6}$$

- The gauge hierarchy problem is solved for $kL \simeq 37$.
- How to fix the size of extra-dimension L?

Radius stabilization with a bulk Higgs

- Goldberger-Wise stabilization mechanism: a real scalar field is employed in the RS1 bulk with appropriate boundary potentials to stabilize the modulus. The minimization of the scalar potential fixs L. hep-ph/9907447

- GW mechanism predicts light scalar Radion: fluctuation of the 5th dim. But LHC has not found any other light scalar except the Higgs boson!

- Can the SM Higgs boson also be the Radion?

- We consider an SU(2) Higgs doublet H as a GW stabilizing field

$$S = \int d^5x \sqrt{-g}\Big\{ -\frac{R}{2} + \left|D_M H\right|^2 - V_B(H) - V_{UV}(H)\delta(y) - V_{IR}(H)\delta(y-L)\Big\}$$

Metric ansatz: $ds^2 = e^{2A(y)}\eta_{\mu\nu}dx^\mu dx^\nu - dy^2$, $M_* = 1$

- We write the y-dependent vev of the Higgs field as:

$$\langle H \rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ \phi_v(y) \end{pmatrix}$$

Radius stabilization with a bulk Higgs doublet

- We assume the scalar potential $V_B(\phi_v)$ of the following form,

$$V_B(\phi_v) = \frac{1}{8}\left(\frac{\partial W(\phi_v)}{\partial \phi_v}\right)^2 - \frac{1}{6}W(\phi_v)^2$$

where the superpotential $W(\phi_v)$ satisfies the following relations:

$$\phi_v' = \frac{1}{2}\frac{\partial W(\phi_v)}{\partial \phi_v}, \qquad A' = -\frac{1}{6}W(\phi_v)$$

$$W(\phi_v)\Big|_{y_i-\epsilon}^{y_i+\epsilon} = V_i(\phi_v)\Big|_{\phi_v=\phi_v(y_i)}, \quad \frac{\partial W(\phi_v)}{\partial \phi_v}\Big|_{y_i-\epsilon}^{y_i+\epsilon} = \frac{\partial V_i(\phi_v)}{\partial \phi_v}\Big|_{\phi_v=\phi_v(y_i)}.$$

- With the brane-localized potentials:

$$V_{UV}(\phi_v) = W(\phi_v) + \frac{\lambda_{UV}}{4k^2}\left(\phi_v^2 - \phi_{UV}^2\right)^2,$$
$$V_{IR}(\phi_v) = -W(\phi_v) + \frac{\lambda_{IR}}{4k^2}\left(\phi_v^2 - \phi_{IR}^2\right)^2,$$

where $\phi_{UV(IR)}$ is the constant value of background vev at $y = 0(\pm L)$ and $\lambda_{UV(IR)}$ is the quartic coupling at the UV (IR) brane.

Radius stabilization with a bulk Higgs doublet

- We consider the following form of superpotential $W(\phi_v)$

$$W(\phi_v) = 6k + (2+\beta)k\phi_v^2 \qquad \text{for} \qquad 0 < y < L$$

where $\beta \equiv \sqrt{4 + \mu_B^2/k^2}$ parameterises the Higgs bulk mass μ_B.

- We get the scalar potential $V_B(\phi_v)$ as

$$V_B(\phi_v) = -6k^2 + \frac{1}{2}\mu_B^2\phi_v^2 - \frac{k^2}{6}(2+\beta)^2\phi_v^4.$$

- The background vev $\phi_v(y)$ and the warp-function $A(y)$ are:

$$\phi_v(y) = \phi_{IR} e^{(2+\beta)k(|y|-L)},$$
$$A(y) = -k|y| - \frac{1}{12}\phi_{IR}^2 e^{-2(2+\beta)kL}\left[e^{2(2+\beta)k|y|} - 1\right],$$

where ϕ_{IR} is the value of background vev at the IR brane:

$$\phi_{IR} = v_{SM}\sqrt{k(1+\beta)}e^{kL}.$$

Radius stabilization with a bulk Higgs doublet

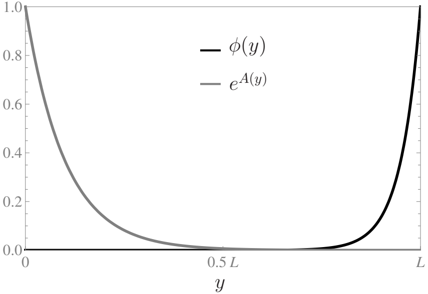
- The brane separation L is fixed by:

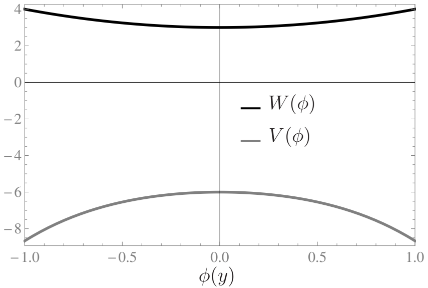
$$kL = \frac{1}{2+\beta} \ln\left(\frac{\phi_{IR}}{\phi_{UV}}\right).$$

- In order to solve the gauge hierarchy problem, i.e.

$$v_{SM} = \frac{\phi_{IR} e^{-kL}}{\sqrt{k(1+\beta)}} \simeq 246 \text{ GeV}, \qquad \text{we need} \quad kL \simeq 37$$

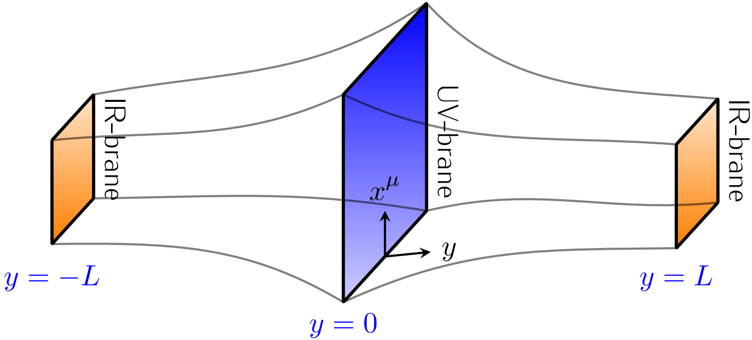
for $\phi_{IR} \simeq \mathcal{O}(M_{Pl})$ and $\beta \sim \mathcal{O}(1)$. This implies $\phi_{UV} \ll \phi_{IR}$.





$\mathbb{Z}_2$ SYMMETRIC GEOMETRY: THE IR–UV–IR MODEL

- WED (RS1-like) models are not symmetric around any point along the extra-dimension $0 \leq y \leq L$ due to non-trivial "warping". Hence no analogue to KK-parity which ensures stable particle!
- We construct a $\mathbb{Z}_2$ symmetric warped extra dimension: IR-UV-IR model
- IR-UV-IR model provides "warped KK-parity". Under this parity all the bulk fields are even or odd, i.e. $\Phi(x,y) \to \Phi^{(\pm)}(x,y)$



SM EWSB by a bulk Higgs doublet

- The 5D action for the gauge sector of the SM:

$$S = -\int d^5x\sqrt{-g}\Big\{\frac{1}{4}F^a_{MN}F^{aMN} + \frac{1}{4}B_{MN}B^{MN} + |D_M H|^2 + \mu_B^2|H|^2$$
$$+ V_{IR}(H)\delta(y+L) + V_{UV}(H)\delta(y) + V_{IR}(H)\delta(y-L)\Big\}$$

$$M, N, \cdots = \underbrace{0, 1, 2, 3}_{\mu,\nu,\cdots}, 5$$

- Due to the $\mathbb{Z}_2$ symmetric geometry (warped KK-parity) the bulk fields in the IR-UV-IR setup separate into even and odd bulk fields:

$$H(x,y) = H^{(+)}(x,y) + H^{(-)}(x,y)$$
$$V_M(x,y) = V_M^{(+)}(x,y) + V_M^{(-)}(x,y)$$

where $V_M \equiv \big(A_M, \underbrace{W_M^{\pm}, Z_M}_{\tilde{V}_M}\big)$.

4D effective theory: after integrating out the 5th dimension

d.o.f. in the 4D effective theory			
5D fields	KK-modes	$n = 0$	$n \neq 0$
Re$H^{(+)}$	$h_n^{(+)}(x)$	✓	✓
Re$H^{(-)}$	$h_n^{(-)}(x)$	✓	✓
Im$H^{(+)}$	$\pi_{Zn}^{(+)}(x)$	✓(GB)	✓
Im$H^{(-)}$	$\pi_{Zn}^{(-)}(x)$	✗(b.c.)	✓
Ch$H^{(+)}$	$\pi_{Wn}^{(+)}(x)$	✓(GB)	✓
Ch$H^{(-)}$	$\pi_{Wn}^{(-)}(x)$	✗(b.c.)	✓
$V_5^{(+)}$	$V_{5n}^{(+)}(x)$	✗(5D g.c.)	✗(5D g.c.)
$V_5^{(-)}$	$V_{5n}^{(-)}(x)$	✗(5D g.c.)	✗(5D g.c.)
$V_\mu^{(+)}$	$V_{\mu n}^{(+)}(x)$	✓	✓
$V_\mu^{(-)}$	$V_{\mu n}^{(-)}(x)$	✗(b.c.)	✓

- Low energy (zero-mode) effective theory contains all the SM d.o.f. with *even parity* and a real scalar with *odd parity*.

Truncated-Inert-Doublet Model

Zero-mode 4D effective action for the SM gauge sector

$$S_{eff} = -\int d^4x \Big\{ \frac{1}{4}\mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu} + \frac{1}{4}\mathcal{Z}_{\mu\nu}\mathcal{Z}^{\mu\nu} + \frac{1}{2}\mathcal{W}^{+}_{\mu\nu}\mathcal{W}^{-\mu\nu} + (\mathcal{D}_\mu H_1)^\dagger \mathcal{D}^\mu H_1 + (\mathcal{D}_\mu H_2)^\dagger \mathcal{D}^\mu H_2 + V(H_1, H_2) \Big\}$$

- $\mathcal{D}_\mu = \partial_\mu - i g_4 \hat{\mathbb{A}}_\mu$ is the usual $SU(2)\times U(1)$ 4D covariant derivative.

$$H_1(x) \equiv e^{i g_4 \widehat{\Pi}(x)} \begin{pmatrix} 0 \\ h_0^{(+)}(x) \end{pmatrix}, \qquad H_2(x) \equiv e^{i g_4 \widehat{\Pi}(x)} \begin{pmatrix} 0 \\ h_0^{(-)}(x) \end{pmatrix}$$

- $\widehat{\Pi}(x) \equiv \widehat{\Pi}_0^{(+)}(x)$ contains even zero-mode Goldstone modes:

$$\widehat{\Pi}(x) \equiv \begin{pmatrix} \frac{\cos^2\theta_W - \sin^2\theta_W}{2\cos\theta_W}\pi_Z & \frac{1}{\sqrt{2}}\pi_W^{+} \\ \frac{1}{\sqrt{2}}\pi_W^{-} & -\frac{1}{2\cos\theta_W}\pi_Z \end{pmatrix}, \qquad \cos\theta_W \equiv \frac{g_4}{\sqrt{g_4^2 + g_4'^2}}, \quad \sin\theta_W \equiv \frac{g_4'}{\sqrt{g_4^2 + g_4'^2}}$$

where $g_4 \equiv \frac{g_5}{\sqrt{2L}}$ and $g_4' \equiv \frac{g_5'}{\sqrt{2L}}$.

Truncated-Inert-Doublet Model

- The scalar potential $V(H_1, H_2)$ can be written as

$$V(H_1,H_2) = -\mu^2|H_1|^2 - \mu^2|H_2|^2 + \lambda|H_1|^4 + \lambda|H_2|^4 + 6\lambda|H_1|^2|H_2|^2$$

$$\mu^2 \equiv -m_0^{2(\pm)} \simeq (1+\beta)m_{KK}^2\delta_{IR}, \qquad \lambda \equiv \lambda_{IR}(1+\beta)^2$$

$$\beta \equiv \sqrt{4+\mu_B^2/k^2}, \quad m_{KK} \equiv ke^{-kL}, \quad \delta_{IR} \equiv \frac{m_{IR}^2}{k^2} - 2(2+\beta)$$

- $V(H_1, H_2)$ is invariant under $[SU(2)\times U(1)_Y]' \times [SU(2)\times U(1)_Y]$; one of them is gauged while the other survived as a global symmetry.

- $[SU(2)\times U(1)_Y]$ is gauged and correspond to the SM gauge symmetry.

- $[SU(2)\times U(1)_Y]$ is spontaneously broken á la Higgs mechanism.

- $V(H_1, H_2)$ is also invariant under various $\mathbb{Z}_2$'s, for example $H_1 \to -H_1$, $H_2 \to -H_2$ and $H_1 \to \pm H_2$.

EWSB OF TRUNCATED–INERT–DOUBLET MODEL

- The scalar potential $V(H_1, H_2)$ has four degenerate global minima.
- We choose the vacuum such that the even parity Higgs field H_1 acquires a vev, whereas the odd parity Higgs field H_2 does not, i.e.

$$v_1^2 \equiv v^2 = \frac{\mu^2}{\lambda}, \qquad v_2 = 0$$

- Let us now consider fluctuations around the vacuum of our choice

$$H_1(x) = \frac{1}{\sqrt{2}} e^{ig_4 \widehat{\Pi}} \begin{pmatrix} 0 \\ v + h \end{pmatrix}, \qquad H_2(x) = \frac{1}{\sqrt{2}} e^{ig_4 \widehat{\Pi}} \begin{pmatrix} 0 \\ \chi \end{pmatrix}$$

- $\widehat{\Pi}$ contains the Goldstone bosons $\pi_{W^\pm}$ and π_Z.
- We choose the unitary gauge in which $\pi_{W^\pm, Z}$ are gauged away, that is they are eaten up by the gauge bosons $W_\mu^\pm$ and Z_μ to get mass.

EWSB OF THE TRUNCATED-INERT-DOUBLET MODEL

Low energy (zero-mode) 4D effective theory

$$S_{eff} = -\int d^4x \Big\{ \frac{1}{4}\mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu} + \frac{1}{4}\mathcal{Z}_{\mu\nu}\mathcal{Z}^{\mu\nu} + \frac{1}{2}\mathcal{W}^{+}_{\mu\nu}\mathcal{W}^{-\mu\nu} + m_W^2 W^+_\mu W^{-\mu}$$
$$+ \frac{1}{2}m_Z^2 Z_\mu Z^\mu + \frac{1}{2}\partial_\mu h \partial^\mu h + \frac{1}{2}m_h^2 h^2 + \frac{1}{2}\partial_\mu \chi \partial^\mu \chi + \frac{1}{2}m_\chi^2 \chi^2 + \frac{\lambda}{4}h^4 + \frac{\lambda}{4}\chi^4$$
$$+ \lambda v h^3 + 3\lambda v h \chi^2 + \frac{3}{2}\lambda h^2\chi^2 + \frac{g_4^2}{2} v W^+_\mu W^{-\mu} h + \frac{1}{4}(g_4^2 + g_4'^2) v h Z_\mu Z^\mu$$
$$+ \frac{g_4^2}{4} W^+_\mu W^{-\mu}(h^2 + \chi^2) + \frac{1}{8}(g_4^2 + g_4'^2) Z_\mu Z^\mu (h^2 + \chi^2) + \mathcal{L}_{\text{gauge self int.}} \Big\}$$

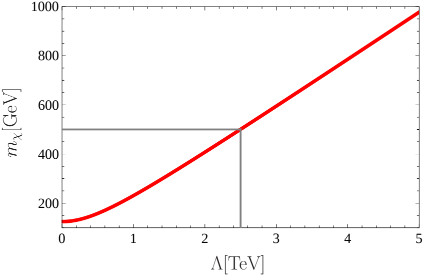
$$m_h^2 = m_\chi^2 = 2\mu^2, \qquad m_W^2 = \frac{g_4^2}{4}\frac{\mu^2}{\lambda}, \qquad m_Z^2 = \frac{1}{4}\left(g_4^2 + g_4'^2\right)\frac{\mu^2}{\lambda}$$

- Our effective theory has a $\mathbb{Z}_2$ discrete symmetry, i.e., all SM fields are even while the scalar χ is odd, $\chi \to -\chi$, the dark matter candidate.
- Note that SM Higgs mass m_h and the dark-Higgs mass m_χ are degenerate at the tree level.

Quantum corrections to scalar masses

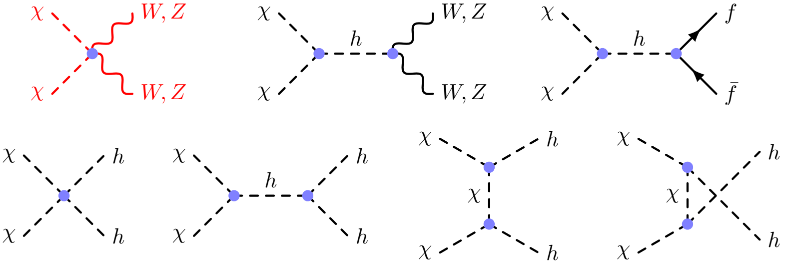
- The Higgs mass m_h and the dark scalar mass m_χ are degenerate at the tree level, $m_h^2 = m_\chi^2 = 2\mu^2$.
- This degeneracy is lifted by the quantum corrections predicted by the effective theory below the KK-mass scale.
- The 1-loop corrected masses for the scalar fluctuations are:

$$m_h^2 = 2\lambda v^2, \qquad m_\chi^2 = 2\lambda v^2 + \frac{3}{4}\frac{\Lambda^2}{\pi^2 v^2} m_t^2, \qquad \Lambda \equiv m_{KK}$$



Dark matter relic abundance

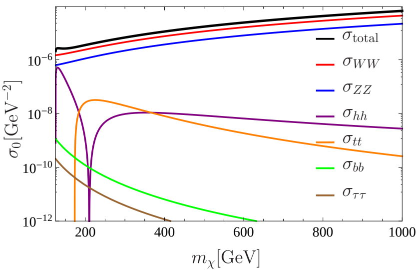
- The Feynman diagrams contributing to the annihilation of dark matter:

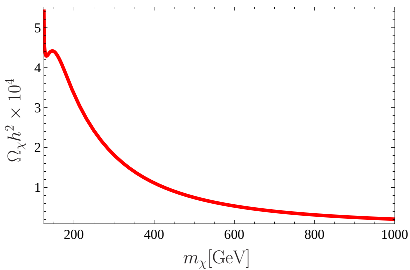


- Note that the $\chi\chi\tilde{V}\tilde{V}$ contact interaction is present in our model since χ is a component of the (truncated) odd $SU(2)$ doublet.
- The $\tilde{V}\tilde{V}$ final states are additionally enhanced by a constructive interference of the contact $\chi\chi\tilde{V}\tilde{V}$ interaction with the s-channel Higgs-exchange diagram.

Dark matter relic abundance

- The total cross section is dominated by WW and ZZ final states.
- Fermionic final states are always irrelevant; even $\chi\chi \to t\bar{t}$ production is very small in comparison to $\chi\chi \to \tilde{V}\tilde{V}$.
- We observe that $\Omega_\chi h^2 \lesssim 10^{-4}$ once the EWPT bound of $m_\chi \gtrsim 500$ GeV is imposed.
- Some other dark matter component is needed within this model to satisfy the Planck measurement, $\Omega_\chi h^2 \sim 0.1$.





Summary

- We provided a radius stabilization mechanism with a bulk Higgs doublet. Superpotential method is employed to get exact analytic b.g. solutions.
- A $\mathbb{Z}_2$ symmetric warped extra-dimensional (IR–UV–IR) model is constructed in order to have a "warped KK-parity".
- Due to the warped KK-parity all bulk fields develop even and odd towers of KK-modes in the 4D effective theory.
- Assuming that the KK-scale is high enough ($m_{KK} \sim \mathcal{O}$(few) TeV), we considered the low energy (zero-mode) effective theory.
- In the low energy (zero-mode) effective theory, we have all the SM fields plus a *dark-Higgs* – the dark matter candidate.
- After the quantum corrections to the tree-level masses; for SM Higgs mass $m_h = 125$ GeV, dark-Higgs mass is 500 GeV $\lesssim m_\chi \lesssim 1200$ GeV.
- For m_χ in the above preferred range, $\Omega_\chi h^2 \lesssim 10^{-4}$ as compared to the current experimental value of ~ 0.1.

Backup slides

SM EWSB by bulk Higgs doublet in IR-UV-IR model

- We KK-decompose the Higgs doublets and the gauge fields as:

$$\mathcal{H}^{(\pm)}(x,y) = \sum_n \mathcal{H}_n^{(\pm)}(x) f_n^{(\pm)}(y),$$

$$\pi_{\tilde{V}}^{(\pm)}(x,y) = \sum_n \pi_{\tilde{V}n}^{(\pm)}(x) a_{\tilde{V}n}^{(\pm)}(y), \qquad V_\mu^{(\pm)}(x,y) = \sum_n V_{\mu n}^{(\pm)}(x) a_{V_n}^{(\pm)}(y)$$

- The wave-functions $f_n^{(\pm)}(y)$ and $a_{V_n}^{(\pm)}(y)$ satisfy

$$-\partial_5\left(e^{4A(y)}\partial_5 f_n^{(\pm)}(y)\right) + \mu_B^2 e^{4A(y)} f_n^{(\pm)}(y) = m_n^2 e^{2A(y)} f_n^{(\pm)}(y)$$

$$-\partial_5\left(e^{2A(y)}\partial_5 a_{V_n}^{(\pm)}(y)\right) = m_{V_n^{(\pm)}}^2 a_{V_n}^{(\pm)}(y).$$

- The jump condition at $y=0$ (UV-brane) and the boundary conditions at $y=\pm L$ (IR-branes) are:

$$\left(\partial_5 - \frac{m_{UV}^2}{2k}\right) f_n^{(+)}(y)\Big|_0 = 0, \qquad f_n^{(-)}(y)\Big|_0 = 0, \qquad \left(\pm\partial_5 - \frac{m_{IR}^2}{2k}\right) f_n^{(\pm)}(y)\Big|_{\pm L} = 0$$

$$\partial_5 a_{V_n}^{(+)}(y)\Big|_0 = 0, \qquad a_{V_n}^{(-)}(y)\Big|_0 = 0, \qquad \partial_5 a_{V_n}^{(\pm)}(y)\Big|_{\pm L} = 0$$

SM EWSB by bulk Higgs doublet in IR-UV-IR model

- It is convenient to write the Higgs doublets in the following form:

$$\begin{pmatrix} H^{(+)} \\ H^{(-)} \end{pmatrix} = e^{ig_5(\Pi^{(+)}\mathbb{1}+\Pi^{(-)}\tau_1)} \begin{pmatrix} \mathcal{H}^{(+)} \\ \mathcal{H}^{(-)} \end{pmatrix}$$

- $\mathcal{H}$ and Π are defined as (the parity indices are suppressed)

$$\mathcal{H}(x,y) \equiv \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ h(x,y) \end{pmatrix}, \qquad \Pi(x,y) \equiv \begin{pmatrix} \frac{\cos^2\theta_5 - \sin^2\theta_5}{2\cos\theta_5}\pi_Z & \frac{1}{\sqrt{2}}\pi_W^+ \\ \frac{1}{\sqrt{2}}\pi_W^- & -\frac{1}{2\cos\theta_5}\pi_Z \end{pmatrix}$$

- We KK-decompose the Higgs doublets and the gauge fields as:

$$\mathcal{H}^{(\pm)}(x,y) = \sum_n \mathcal{H}_n^{(\pm)}(x) f_n^{(\pm)}(y),$$

$$\pi_{\tilde{V}}^{(\pm)}(x,y) = \sum_n \pi_{\tilde{V}n}^{(\pm)}(x) a_{\tilde{V}n}^{(\pm)}(y), \qquad V_\mu^{(\pm)}(x,y) = \sum_n V_{\mu n}^{(\pm)}(x) a_{V_n}^{(\pm)}(y)$$

Low energy (zero-mode) 4D effective theory

- We use the above KK-decomposition and integrate over the extra-dim y to get 4D effective theory.
- Assuming KK-scale is high enough, i.e. $m_{KK} \sim \mathcal{O}(\text{few})$ TeV; we consider an effective theory with only the zero-modes ($m_0 \ll m_{KK}$).
- The zero-mode wave-functions for Higgs doublets are:

$$f_0^{(\pm)}(|y|) \simeq \sqrt{k(1+\beta)} e^{kL} e^{(2+\beta)k(|y|-L)}, \qquad \beta \equiv \sqrt{4+\mu_B^2/k^2}$$

- The zero-mode wave-functions for gauge fields with our choice of b.c.:

$$a_{V_0}^{(-)}(y) = 0, \qquad a_{V_0}^{(+)}(y) = 1/\sqrt{2L}$$

- Which implies the odd zero-mode gauge fields $V_{0\mu}^{(-)}(x)$ and the odd scalar modes $\pi_{\tilde{V}_0}^{(-)}(x)$ are not present in the effective 4D theory.