Extending the Matrix Element Method beyond the Born approximation: Calculating event weights at next-to-leading order accuracy.

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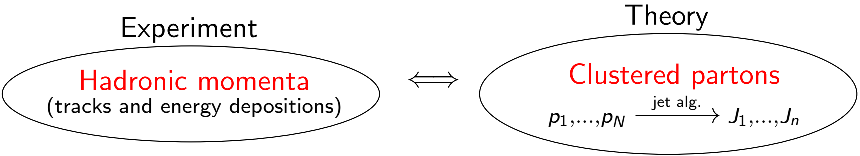
Jet Event Weight: Definition & Application

Jet



Jet event

Observed (jet-) momenta $J_1, \ldots, J_n$ from $P_A + P_B \rightarrow J_1 + \ldots + J_n + X$

Jet event weight

Interpreting differential cross section as probability density to measure a specific event: $\rho \propto \frac{d\sigma_{AB\rightarrow n}}{d^4J_1 \ldots d^4J_n}$

- Construct more inclusive (jet-) observables
- Generate unweighted events according to ρ
- Used in likelihood analysis methods (Matrix Element Method)

Matrix Element Method (MEM) in a nutshell [Kondo '88,'91]

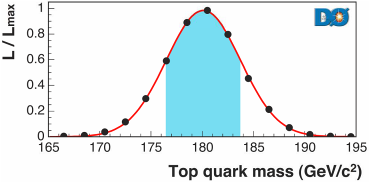
Extraction of model parameters Ω from data by maximizing a likelihood $\propto$ differential cross section ($\propto |\mathcal{M}^{LO}|^2$):

Likelihood for Ω for set of events e.g. $\vec{x}_i = (J_1, \ldots, J_n)_i$

$$\mathcal{L}^{LO}(\Omega) = \prod_i \frac{1}{\sigma^{LO}(\Omega)} \int d\vec{y} \frac{d\sigma^{LO}(\Omega)}{d\vec{y}} \underbrace{W(\vec{x}_i, \vec{y})}_{\substack{\text{transfer function,} \\ \text{here: } =\delta(\vec{x}_i - \vec{y})}} = \prod_i \frac{1}{\sigma^{LO}(\Omega)} \frac{d\sigma^{LO}(\Omega)}{d\vec{x}_i}$$

Maximizing wrt Ω yields estimator $\widehat{\Omega}$: $\mathcal{L}^{LO}(\widehat{\Omega}) = \sup_{\Omega} \mathcal{L}^{LO}(\Omega)$

All information from event used $\Longrightarrow$ most efficient estimator!



e.g. top mass measurement at Tevatron [D0: Nature 429, 638], [CDF: PRD 50, 2966 (1994)] based on $O(40)$ events!

Likelihood at NLO and 3 problems

$$\mathcal{L}^{NLO}(\Omega) = \prod_i \frac{1}{\sigma^{NLO}_{\text{n-jet}}(\Omega)} \left(\frac{d\sigma^{NLO}_{n\to\text{n-jet}}(\Omega)}{dJ_1 \dots dJ_n} + \frac{d\sigma^{NLO}_{n+1\to\text{n-jet}}(\Omega)}{dJ_1 \dots dJ_n} \right)\Bigg|_{\text{event } i}$$

Born+virtual and real contribution separately IR divergent

NEED: Point-wise cancelation

→ both contributions must be evaluated for same jet momentum

Real (one recombination $J_i = \widetilde{J}_i(p_1, \ldots, p_{n+1})$)

$$\frac{\sigma^{NLO}_{n+1\to\text{n-jet}}(\Omega)}{dJ_1 \ldots dJ_n} = \int dR_{n+1} \frac{d\sigma^{NLO}_R(\Omega)}{dR_{n+1}} \prod_{i=1}^{n} \delta(\widetilde{J}_i(p_1, \ldots, p_{n+1}) - J_i)$$

Integration over δ-function numerically not feasible!

NEED: factorisation of phase space

$\rightarrow$ Integration trivial:

$$dR_{n+1}(p_1, \ldots, p_{n+1}) = dR_n(\widetilde{J}_1, \ldots, \widetilde{J}_n) dR_{\text{unres}}(\Phi)$$

$$\Rightarrow \quad dR_{n+1}(p_1, \ldots, p_{n+1}) \prod_{i=1}^{n} \delta(\widetilde{J}_i - J_i) = dR_{\text{unres}}(\Phi)\Big|_{\widetilde{J}_i = J_i}$$

$dR_{\text{unres}}(\Phi)$ generates only partonic configurations that result in given jet event (inverted jet algorithm)

Born+virtual (no recombination $J_i = p_i = \widetilde{J}_i$)

$$\frac{\sigma^{NLO}_{n\to\text{n-jet}}(\Omega)}{dJ_1 \dots dJ_n} = \int dR_n \frac{d\sigma^{NLO}_{B+V}(\Omega)}{dR_n} \prod_{i=1}^{n} \delta(p_i - J_i) = \frac{\sigma^{NLO}_{B+V}(\Omega)}{dJ_1 \dots dJ_n}$$

Born+virtual matrix elements only defined for Born kinematics

NEED: clustered jets obeying Born kinematics

$\rightarrow$ on-shell condition and momentum conservation:

$$\widetilde{J}_i^2 = m_i^2 \quad \text{and} \quad p_1 + \dots + p_{n+1} = \widetilde{J}_1 + \widetilde{J}_n$$

not possible with $2 \rightarrow 1$ clustering/recombination

$3 \to 2$ clustering [Catani,Seymour '97], [Catani, Dittmaier,Seymour,Trocsanyi '02]

$3 \to 2$ clusterings from dipole subtraction method meet all 3 requirements at the same time

$\Downarrow$

Using $3 \to 2$ jet algorithm instead of $2 \to 1$ solves our 3 problems

Jet event weight at NLO

Phase space factorisation allows to define an event weight (differential jet cross section) at NLO

$$\frac{d\sigma^{NLO}_{\text{n-jet}}(\Omega)}{dJ_1 \dots dJ_n} = \frac{d\sigma^{NLO}_{B+V}(\Omega)}{dJ_1 \dots dJ_n} + \underbrace{\int dR_{\text{unres}}(\Phi)}_{\text{3dim integration}} \frac{d\sigma^{NLO}_{R}(\Omega)}{dp_1 \dots dp_{n+1}}$$

Mutual cancelation of IR-divergences from the virtual and the real part has to be carried out by a suitable method (e.g. phase space slicing)

Validation

Calculate differential jet distributions in NLO accuracy using traditional approach (parton level MC + 3 $\to$ 2 jet alg.)

1. Compare with distributions obtained from $\frac{d\sigma^{NLO}_{\text{n-jet}}(\Omega)}{dJ_1 \dots dJ_n}$
2. Compare with histogrammed unweighted events generated according to $\rho = \frac{d\sigma^{NLO}_{\text{n-jet}}(\Omega)}{dJ_1 \dots dJ_n}$

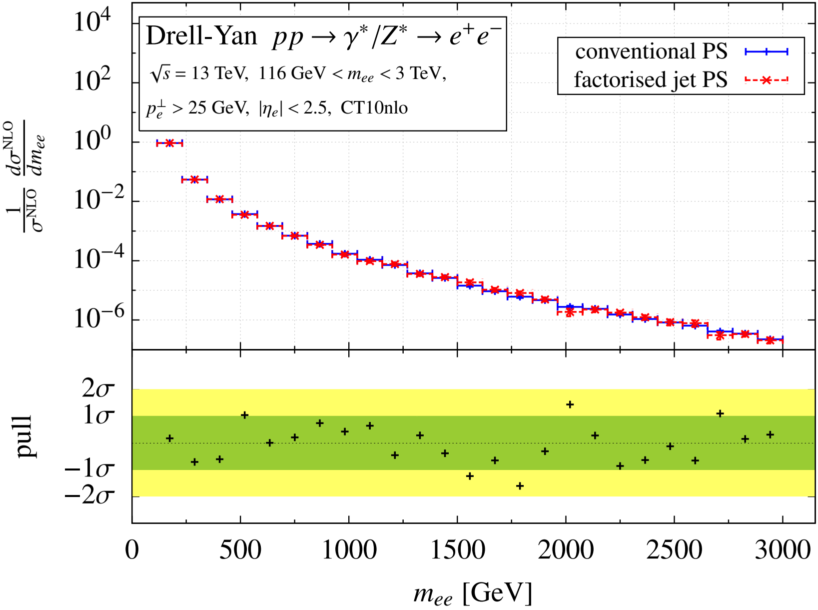
Sample processes:

Drell-Yan $pp \to e^+e^-$	top pair production $e^+e^- \to t\bar{t}$ (no decay)
hadrons in initial state	massive colored particles in final state

Veto on additional jet emission, no resolved additional jet!

Although simple cover most relevant cases

Validation 1: Phase space generation ($pp \to e^+e^-$)



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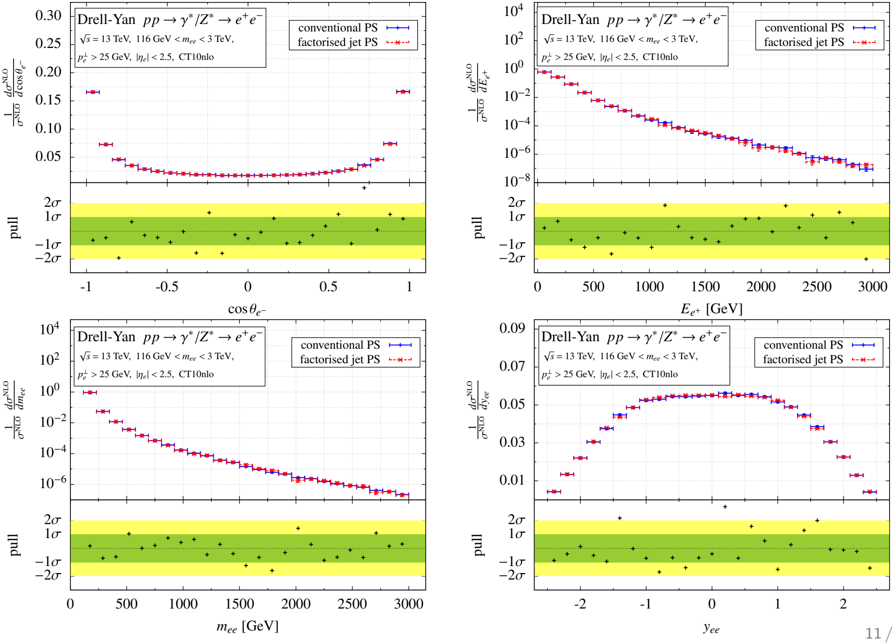

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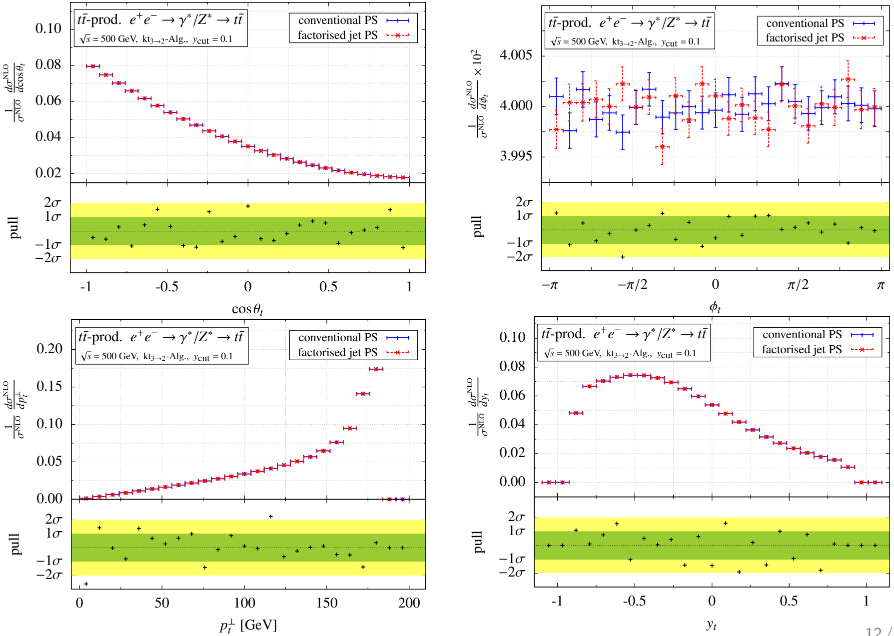
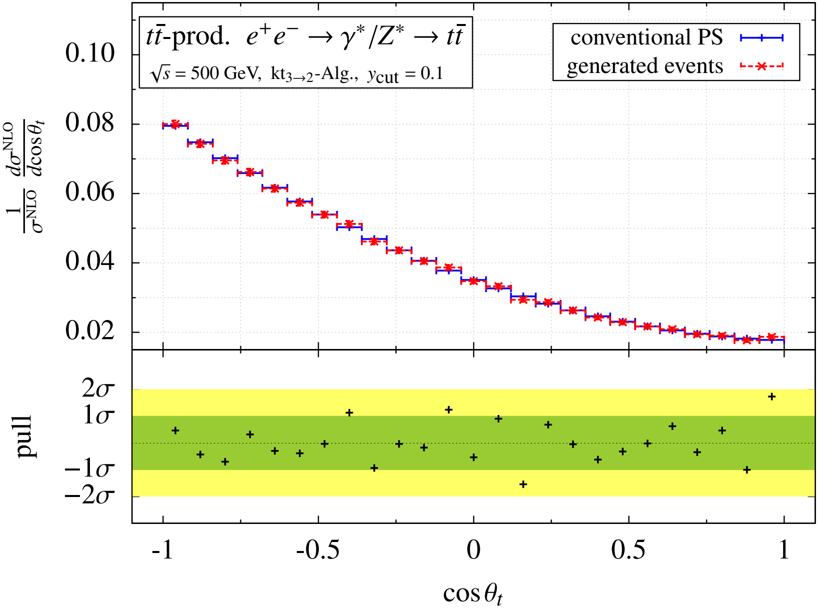


Validation 2: Unweighted events ($e^+e^- \to t\bar{t}$)



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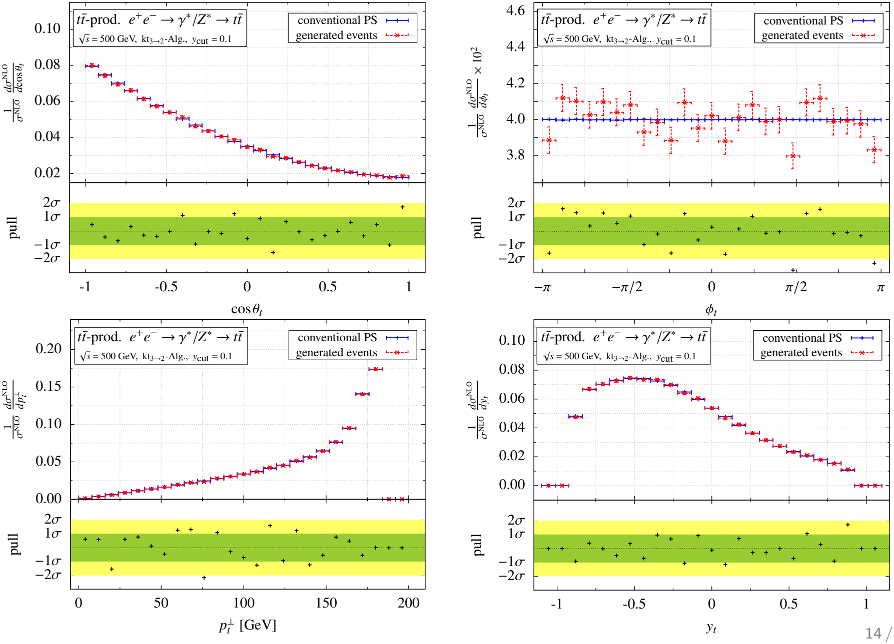


Matrix Element Method at NLO

(example: $e^+e^- \to t\bar{t}$ without additional jet)

Toy experiment: Generated sample of N unweighted NLO $t\bar{t}$ events

$$\vec{x}_i = (\cos\theta_t, \phi_t, \cos\theta_{\bar{t}}, \phi_{\bar{t}}) \quad \text{with } \Omega = m_t = m_t^{\text{true}} = 174 \text{ GeV}$$

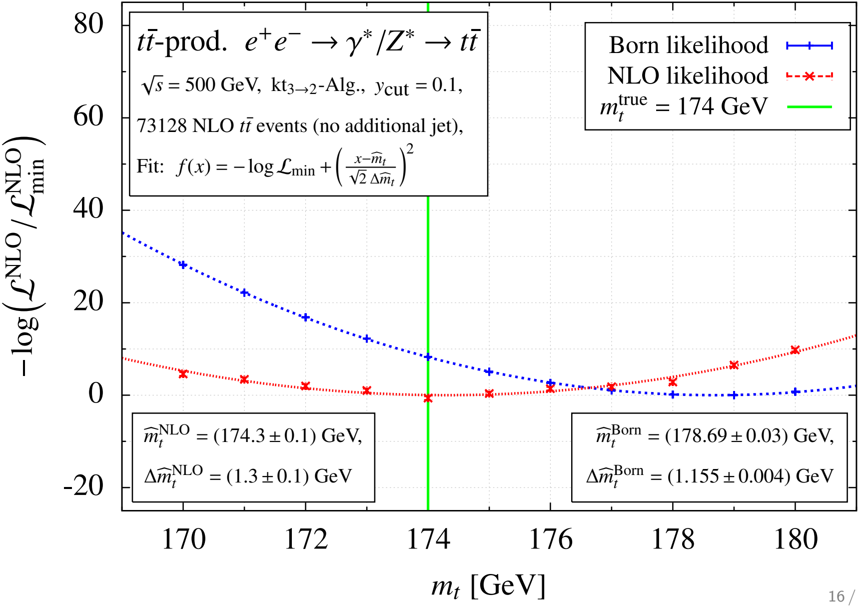
NLO likelihood function for event sample

$$\mathcal{L}^{NLO}(m_t) = \prod_i^N \mathcal{L}^{NLO}(\vec{x}_i|m_t) = \left(\frac{\beta_t}{32\pi^2\sigma_{t\bar{t}}^{NLO}(m_t)}\right)^N \prod_i^N \frac{d\sigma_{t\bar{t}}^{NLO}(m_t)}{dJ_t dJ_{\bar{t}}}\Big|_{\text{event } i}$$

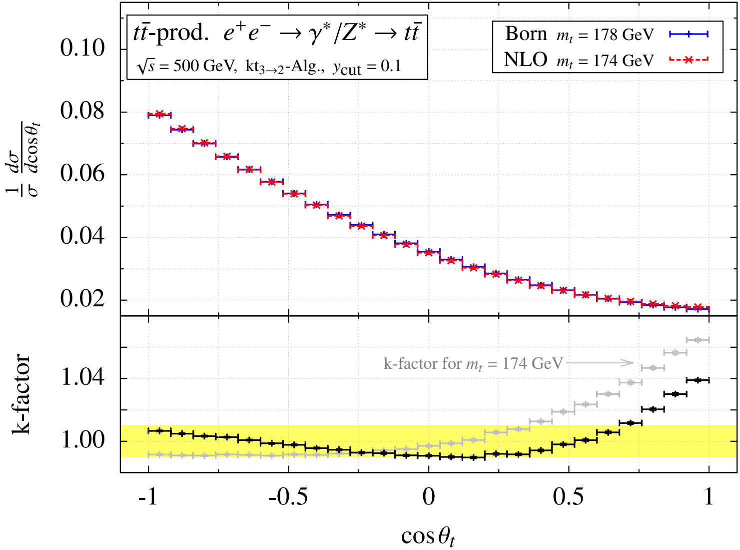
Find minimum of negative logarithm of NLO likelihood ("Log-Likelihood") to obtain estimator $\widehat{m}_t$ for top mass

$$-\log\mathcal{L}^{NLO}(\widehat{m}_t) = \inf_{m_t}\left(-\log\mathcal{L}^{NLO}(m_t)\right)$$

MEM at NLO: top-quark mass extraction via parabola fit



NLO with $m_t = 174$ GeV vs. Born with $m_t = 178$ GeV



NLO corrections small but large effects in MEM anyway!

Conclusion

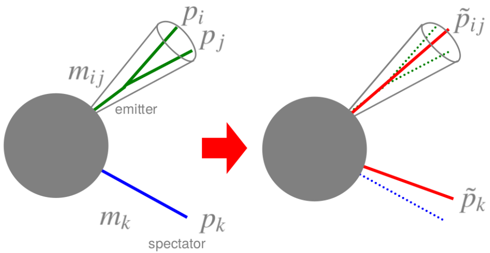
- $3 \to 2$ jet clustering algorithm:
  - Unique mapping of real corrections onto Born kinematics
- Evaluation of event weights for jet events in NLO accuracy
- Generation of unweighted events at NLO
- Application of MEM at NLO
  - Extraction of m_t with NLO likelihood from NLO $t\bar{t}$ events: Perfect agreement with input value!
  - Extraction with Born likelihood: Large deviation from input value possible! (despite small NLO corrections)
  - Renormalization scheme well-defined in MEM at NLO

Outlook: MEM at NLO for top-pair, single top, ... at LHC

BackUp: Modified clustering

[Catani,Seymour '97], [Catani, Dittmaier,Seymour,Trocsanyi '02]

$$(p_i, p_j, p_k) \to (\widetilde{p}_{ij}, \widetilde{p}_k) \equiv (J_{ij}, J_k)$$



$$J_{ij} + J_k = p_i + p_j + p_k$$

and $$J_{ij}^2 = m_{ij}, \;\; J_k = m_k$$

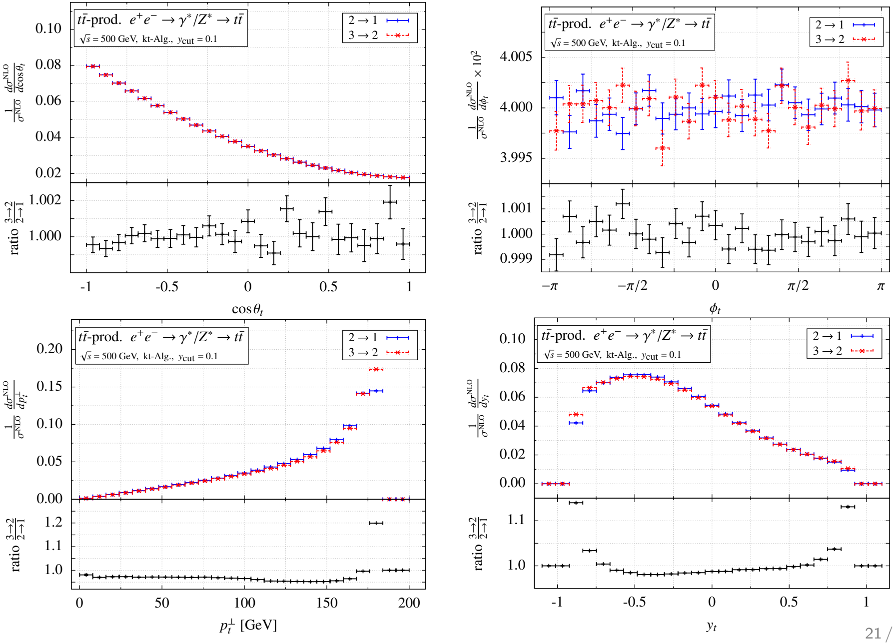
Phase space factorises: $$dR_{n+1}(p_i, p_j, p_k) = dR_n(J_{ij}, J_k) dR_{\text{unres}}(\Phi)$$

$dR_{\text{unres}}(\Phi)$ generates all p_i, p_j, p_k with $(p_i, p_j, p_k) \stackrel{!}{\to} (J_{ij}, J_k)$

BackUp: $3 \to 2$ as an augmented $2 \to 1$

- Use resolution criterium of the $2 \to 1$ algorithm to pick final state particle to be clustered with final state particle or beam ("emitter")
- Choose final state particle or beam as "spectator"
- 4 different types of mappings
  (emitter,spectator) = (final,final), (final,initial), (initial, initial), (initial,final)
- Respective clusterings for massless and massive particles already worked out in Catani-Seymour dipole subtraction method [Catani,Seymour '97], [Catani, Dittmaier,Seymour,Trocsanyi '02] (let's use those!)

BackUp: Impact of $3 \to 2$ wrt $2 \to 1$ for $e^+e^- \to t\bar{t}$



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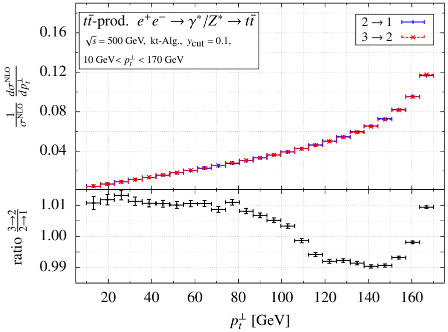
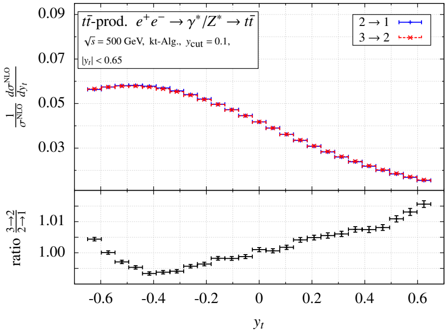
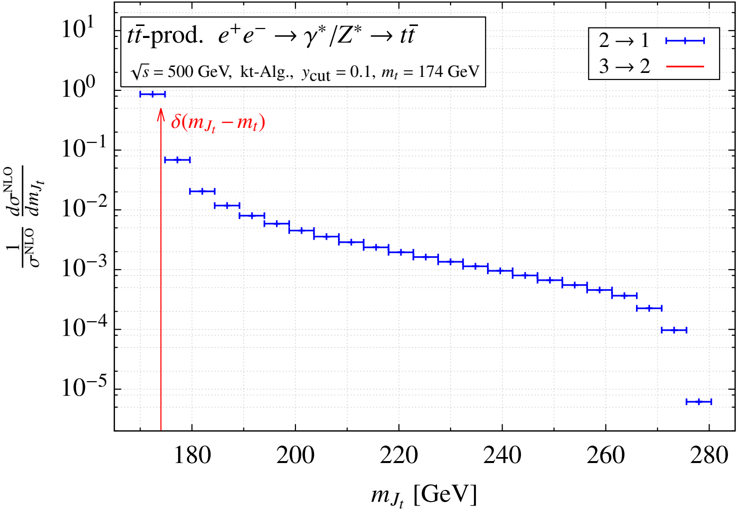
$p_t^\perp$ and y_t distributions with cuts to avoid phase space boundaries



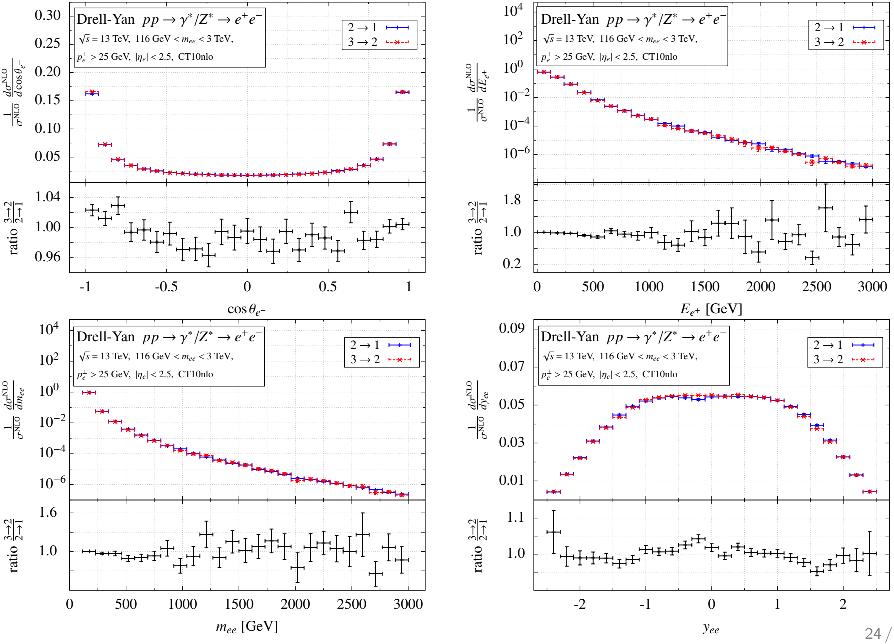


BackUp: Impact of $3 \to 2$ wrt $2 \to 1$ for $e^+e^- \to t\bar{t}$

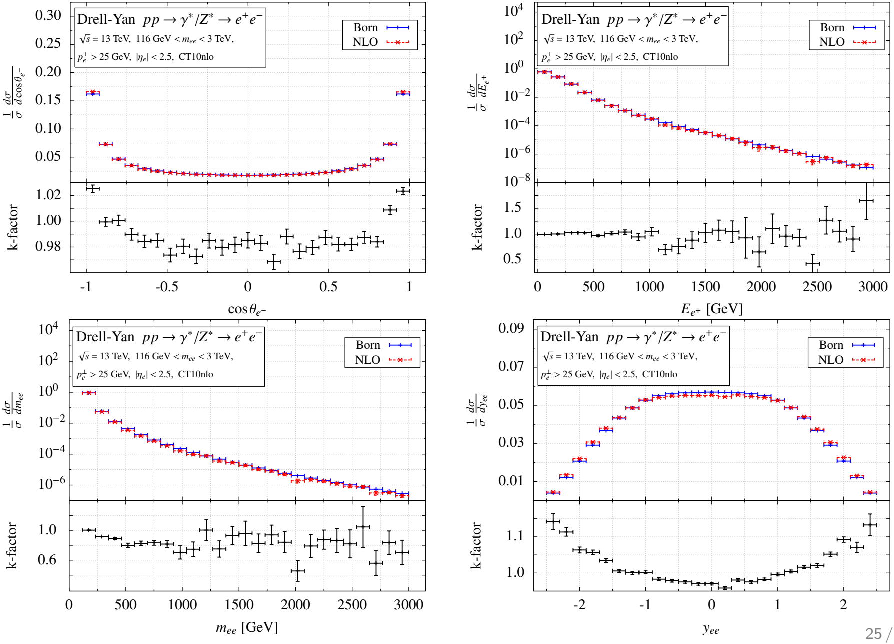
Mass distribution of top jet from $2 \to 1$ clustering



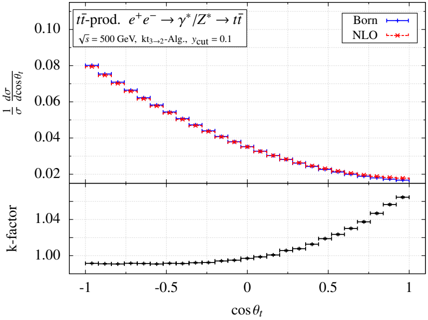
BackUp: Impact of $3 \to 2$ wrt $2 \to 1$ for $pp \to e^+e^-$

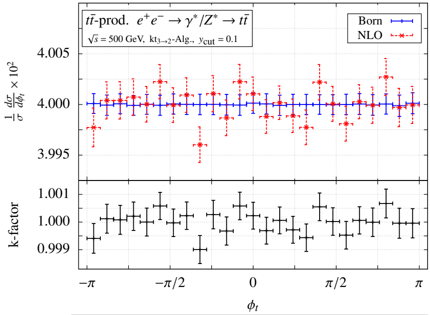


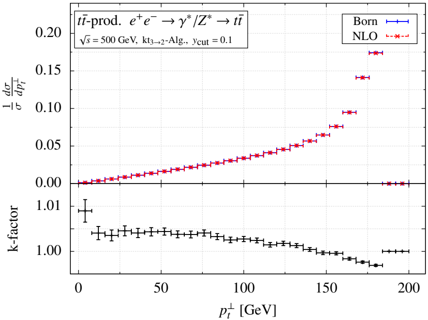
BackUp: Impact of NLO: k-factors for $pp \to e^+e^-$

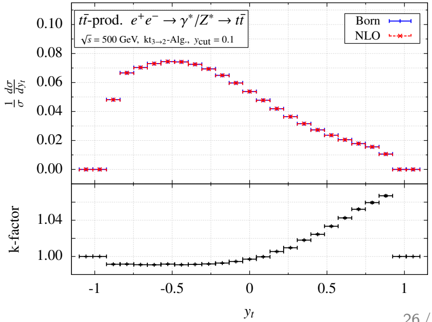


BackUp: Impact of NLO: k-factors for $e^+e^- \to t\bar{t}$

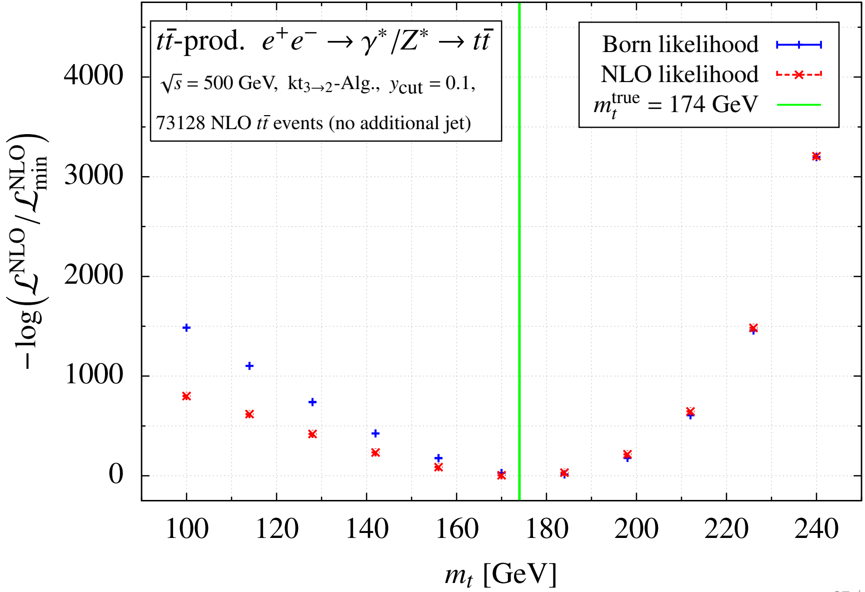




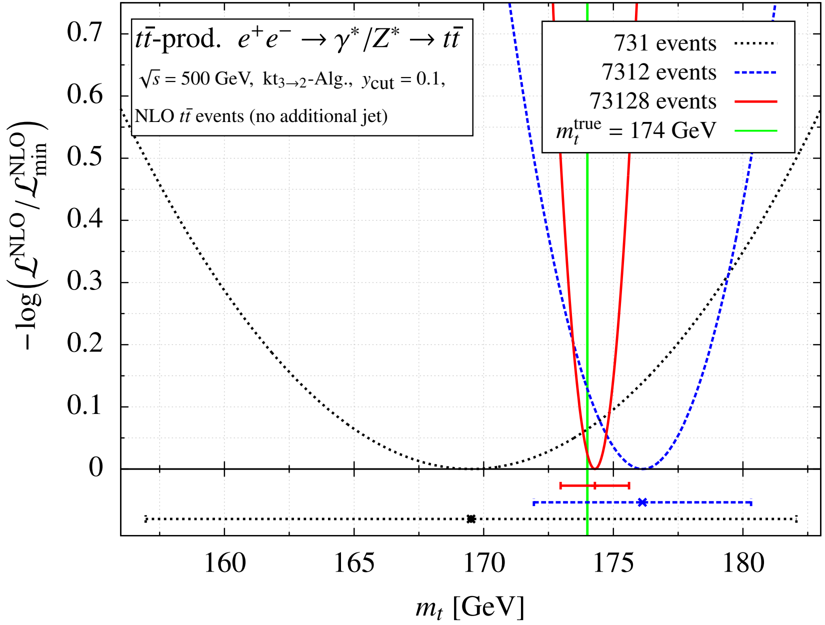




BackUp: MEM at NLO: Log-likelihood as a function of m_t



BackUp: Consistency of MEM at NLO



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