NLO corrections to hard process in Parton Shower MC – KrkNLO method

S. JADACH

Contributions by: W. Płaczek, M. Sapeta, A. Siódmok, and M. Skrzypek

Institute of Nuclear Physics PAN, Kraków, Poland

Partly supported by the grants of *Narodowe Centrum Nauki* **DEC-2011/03/B/ST2/02632 and UMO-2012/04/M/ST2/00240**

To be presented at Ustroń, September 2015

INTRODUCTION: from DGLAP to parton shower MC

- Early activity (2004-06) on Patron Shower Monte Carlo and NLO QCD started with solving exactly LO and NLO DGLAP evolution eqs. using Markovian methods, MMC programs:
  – *Acta Phys.Polon.B37:1785*, [arXiv:hep-ph/0603031]
  – *Acta Phys.Polon.B38:115*, [arXiv:0704.3344]
  – *Comput.Phys.Commun.181:393*,[arXiv:0812.3299]
- These MMCs were also capable to evolve CCFM evol. + DGLAP
- MMCs were used to xcheck CMC series of programs (2005-07).
  – *Comput.Phys.Commun.175:511*, [arXiv:hep-ph/0504263]
  – *Comput.Phys.Commun.180:6753*,[arXiv:hep-ph/0703281]
- CMCs implement the same evolution with constrained/predefined final x, an alternative to backward evolution in the PS MC, aiming at better control (NLO) of the distribs. generated by LO PS MC.
- CMCs were for single ladder/shower, without hard process, with exclusive LO kernels, optionally inclusive NLO kernels.

Introduction2: from DGLAP to parton shower MC

- Two CMC modules and hard process ME were combined into complete PSMC for Drell-Yan process, see for example:
  – *Acta Phys.Polon.B38(2007)2305*,
  – *Acta Phys.Polon.B43(2012)2067* ,
  unfortunately not upgraded with realistic PDFs and kinematic.
- However, this kind of PS MC has been instrumental in testing new ideas on implementing:
  1. NLO corrections in the exclusive evolution kernels in the initial state ladders/showers many times,
  2. NLO corrections to hard process just once (a simpler alternative to MC@NLO and POWHEG)

  thanks to perfect numerical and algebraic control over LO distributions.
- ...see next slides.

Introduction3: NLO corrections to PS MC

- The problem of including NLO corrs. in exclusive form into evolution (kernels) in the (initial state) ladder/shower was never addressed before.
- Except of statements that it is for sure unfeasible:)
- First solution, albeit limited to non-singlet evol. kernels, was proposed and tested numerically in:
  – *Acta Phys.Polon. B40(2009)2071*, [arXiv:0905.1399],
  – *Proc. of RADDCOR 2009, [arXiv:1002.0010]*
- ... using NLO kernels in exclusive form calculated from the scratch in the Curci-Furmanski-Petronzio (CFP) framework.
  Non-singlet 2-real kernels were presented in:
  – *JHEP 1108(2011)012*, [arXiv:1102.5083]
- Simplified and faster scheme reported (numerical tests) in:
  – *Nucl.Phys.Proc.Suppl. 205-206(2010)295* , [arXiv:1007.2437]

Introduction4: NLO corrections to PS MC

- Even simpler and faster scheme of NLO-correcting PS MC (single initial state ladder) reported in Ustron 2013 Proceedings:
  – *Acta Phys.Polon. B44 (2013) 11, 2179-2187*, [arXiv:1310.6090]
- Also singlet evolution kernels are now almost complete (unpublished).
- It is a major problem to include consistently virtual corrections to exclusive kernels starting from CFP scheme.
- First solution was formulated (unpublished) exploiting recalculated virtual corrections in CFP scheme to non-singlet kernels:
  – *Acta Phys.Polon. B44 (2013) 11, 2197* , [arXiv:1310.7537]
- The above breakthrough is important but points to:
  (i) need of better understanding of the MC distributions in the PS MC,
  (ii)especially their kinematics, definition of the evolution variable etc.
- For the time being this area of the development is not very active:(

NLO corrections to hard process - recent activity

KrkNLO project of adding NLO corrections to DY hard process [arXiv:1111.5368]

Implemented on top of SHERPA and HERWIG (instead of two CMCs). Comparisons of KrkNLO numerical results with NLO calculations of MCFM (fixed order NLO), MC@NLO and POWHEG, for Drell-Yan process.

Preliminary earlier developments:

1. Methodology of the KrkNLO for DY process was defined in Ustron 2011 Proc., but without numerical test:
   – *Acta Phys.Polon. B42 (2011) 2433* , [arXiv:1111.5368]
2. Numerical validation of KrkNLO on top of Double-CMC PS was shown in:
   – *Acta Phys.Polon. B43 (2012) 2067* , [arXiv:1209.4291]
3. Most complete discussion of the KrkNLO scheme, introducing PDFs in the MC factorization scheme, was provided in:
   – *Phys.Rev. D87 (2013) 3, 034029* , [arXiv:1103.5015],
   but MC implementation still on top of not so realistic Double-CMC PS.

Finally, recent arXiv:1503.06849 (to appear in JHEP) 50 pages, 14 figures:
S. Jadach, W. Placzek, S. Sapeta, A. Siodmok and M. Skrzypek, "Matching NLO QCD with parton shower in Monte Carlo scheme - the KrkNLO method,"

NLO weight for re-weighting LO parton shower events

for the $q\bar{q}$ channel in the KrkNLO, in terms of Sudakov variables α and β

$$d\sigma^{\rm NLO}_{n_F n_B} = \Big(1 + \Delta_{VS} + \sum_{i=1}^{n_F} W^{[1]}_{q\bar{q}}(\tilde{\alpha}^F_i, \tilde{\beta}^F_i) + \sum_{j=1}^{n_B} W^{[1]}_{q\bar{q}}(\tilde{\alpha}^B_j, \tilde{\beta}^B_j)\Big) d\sigma^{\rm LO}_{n_F n_B},$$

$$W^{[1]}_{q\bar{q}} = \frac{d^5\bar{\beta}_{q\bar{q}}}{d^5\sigma^{\rm LO}_{q\bar{q}}} = \frac{d^5\sigma^{\rm NLO}_{q\bar{q}} - d^5\sigma^{\rm LO}_{q\bar{q}}}{d^5\sigma^{\rm LO}_{q\bar{q}}}, \qquad \Delta^{q\bar{q}}_{VS} = \frac{\alpha_s}{2\pi} C_F \left[\frac{4}{3}\pi^2 - \frac{5}{2}\right], \qquad \Delta^{qg}_{VS} = 0.$$

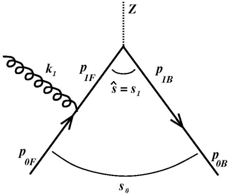
$$d^5\sigma^{NLO}_{q\bar{q}}(\alpha,\beta,\Omega) = \frac{C_F\alpha_s}{\pi}\,\frac{d\alpha\, d\beta}{\alpha\beta}\,\frac{d\varphi}{2\pi}\,d\Omega\left[\frac{d\sigma_0(\hat{s},\theta_F)}{d\Omega}\,\frac{(1-\beta)^2}{2} + \frac{d\sigma_0(\hat{s},\theta_B)}{d\Omega}\,\frac{(1-\alpha)^2}{2}\right],$$

$$d^5\sigma^{\rm LO}_{q\bar{q}}(\alpha,\beta,\Omega) = d^5\sigma^F_{q\bar{q}} + d^5\sigma^B_{q\bar{q}} = \frac{C_F\alpha_s}{\pi}\,\frac{d\alpha\, d\beta}{\alpha\beta}\,\frac{d\varphi}{2\pi}\,d\Omega\,\frac{1+(1-\alpha-\beta)^2}{2}\,\frac{d\sigma_0}{d\Omega}(\hat{s},\hat{\theta}),$$

- Kinematics and LO PS differential distribution $\sigma^{\rm LO}_{n_F n_B}$ to be defined below.
- Important point: As pointed out in [arXiv:1209.4291], for getting complete NLO corrections to the hard process, it is enough to retain in the above sums over gluons $\sum_j$ only a single term, the one with the maximum k_T^2 from one of the two showers.
- In the case of the backward evolution algorithm and k_T-ordering, retained gluon is just the one which was generated first.
- This exploits Sudakov suppression as POWHEG, but no need of truncated shower for angular ordering.

Kinematics

Full coverage of the hard gluon phase space by LO PSMC is essential for KrkNLO!

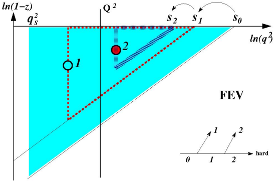


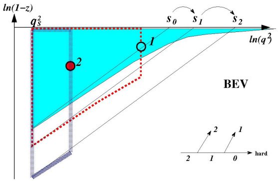
$$\alpha_1 = \frac{2k_1 \cdot p_{0_B}}{s_0} = \frac{2k_1^+}{s_0}, \quad \beta_1 = \frac{2k_1 \cdot p_{0_F}}{s_0} = \frac{2k_1^-}{s_0}, \quad \alpha_1 + \beta_1 \le 1, \quad \alpha_1, \beta_1 \ge 0,$$

$$z_1 = 1 - \alpha_1 - \beta_1\,, \quad k_{1T}^2 = \alpha_1 \beta_1 s_0\,, \quad y_1 = \frac{1}{2} \ln \frac{\alpha_1}{\beta_1}, \quad s_0 = 2p_{0_F} p_{0_B},$$

$$\text{Evolution variables} \quad (k_T - like) \quad q^2_{1_F} = s_0(\alpha_1 + \beta_1)\beta_1\,, \quad q^2_{1_B} = s_0(\alpha_1 + \beta_1)\alpha_1\,.$$

Phase space limits in forward (FEV) and backward (BEV) evolution for up to 2 emissions:





Luckily in modern LO PS MCs like Sherpa and HERWIG full phase space coverage is implemented, in spite of more complicated phase space in BEV parametrization.

Compatibility of forward (FEV) and backward (BEV) distribs.

from LO PSMC was analyzed up to NLO level for the 1st time

Forward evol.

$$\sigma^{\rm LO}_{\rm MC} = \int dx_F dx_B d\Omega \sum_{n_F=0}^{\infty} \sum_{n_B=0}^{\infty} \int d\sigma^{\rm LO}_{n_F n_B},$$

$$d\sigma^{\rm LO}_{n_F n_B} = \prod_{i=1}^{n_F} \prod_{j=1}^{n_B} \Big\{ \int d^3\rho_i^F \theta_{q^2_{i-1}>q^2_i>q^2_s} e^{-S_F(q^2_{i-1},q^2_i)} \Big\} \Big\{ \int d^3\rho_j^B \theta_{q^2_{j-1}>q^2_j>q^2_s} e^{-S_B(q^2_{j-1},q^2_j)} \Big\}$$
$$\times e^{-S_F(q^2_{n_F},q^2_s)} e^{-S_B(q^2_{n_B},q^2_s)} \frac{d\sigma}{d\Omega}(sx_F x_B,\hat\theta) \frac{1}{Z_{n_F}} D^F_{\rm MC}\Big(q^2_s, \frac{x_F}{Z^F_{n_F}}\Big) \frac{1}{Z_{n_B}} D^B_{\rm MC}\Big(q^2_s, \frac{x_B}{Z^B_{n_B}}\Big),$$

$$d^3\rho_i^F = d^3\rho_i^F(s_{ij}) = \frac{d\tilde\beta_i d\tilde\alpha_i}{\tilde\beta_i(\tilde\alpha_i+\tilde\beta_i)} \frac{d\phi_i}{2\pi} \bar{\mathcal{P}}(1-\tilde\alpha_i-\tilde\beta_i)\, \theta_{\tilde\alpha_i>0}\, \theta_{\tilde\alpha_i+\tilde\beta_i<1}$$
$$= \frac{dq_i^2 dz_i}{q_i^2} \frac{d\phi_i}{2\pi} \theta_{(1-z_i)^2 s_{ij}>q_i^2} \mathcal{P}(z_i) = \frac{dq_i^2}{q_i^2} \frac{d\phi_i}{2\pi} dz_i \frac{\bar{\mathcal{P}}(z_i)}{1-z_i} \theta_{(1-z_i)^2 s_{ij}>q_i^2},$$

$$S_F(q_b^2,q_a^2) = S_F(s_{ij}|q_b^2,q_a^2) \equiv \int_{q_a^2<q_i^2<q_b^2} d^3\rho_i^F(s_{ij}),$$

Backward evolution

$$d\sigma^{\rm LO}_{n_F n_B} = \frac{d\sigma}{d\Omega}(sx_F x_B,\hat\theta) \prod_{i=1}^{n_F} \Big\{ d^3\omega_i^F\, \theta_{q^2_{i-1}>q^2_i} \Big\} \prod_{j=1}^{n_B} \Big\{ d^3\omega_j^B\, \theta_{q^2_{j-1}>q^2_j} \Big\}$$
$$\times e^{-\Delta_F(x^F_{n_F}|q^2_{n_F},q^2_s)} e^{-\Delta_B(x^B_{n_B}|q^2_{n_B},q^2_s)} D^F(\hat s, x_F)\, D^B(\hat s, x_B)\, dx_F dx_B d\Omega,$$
$$x_i^F = x_F/Z_i^F,\; x_j^B = x_B/Z_j^B,\; \hat s = s x_F x_B,$$

$$d^3\omega_i^H = \frac{dq_i^2 dz_i}{q_i^2} \frac{d\phi_i}{2\pi} \mathbb{K}_{\rm MC}(x_{i-1}|z_i,q_i^2)\, e^{-\Delta_{\rm MC}(x_{i-1}|q_i^2,q_{i-1}^2)},$$

$$\mathbb{K}_{\rm MC}(x^*|z,q^2) \equiv \mathcal{P}(z)\, \theta_{(1-z)^2 sx^*/z>q^2} \frac{\overline{D}_{\rm MC}(sx^*/z|q^2,x^*/z)}{\overline{D}_{\rm MC}(sx^*|q^2,x^*)},$$

$$\Delta_{\rm MC}(x^*|q^2_{j-1},q^2_j) \equiv \int_{q_j^2}^{q^2_{j-1}} \frac{dq^2}{q^2} \int_{x^*}^1 \frac{dz}{z} \mathbb{K}_{\rm MC}(x^*|z,q^2),$$

Formal algebraic proof of NLO-compatibility between FEV and BEV is based on 2 elements:

1. Multiple use of identity eliminating/introducing BEV form-factor and ratios of PDFs:

$$e^{-S_{\rm MC}(\hat s|q_b^2,q_a^2)} = e^{-\Delta_{\rm MC}(x|q_b^2,q_a^2)} \frac{\overline{D}_{\rm MC}(\hat s|q_b^2,x)}{\overline{D}_{\rm MC}(\hat s|q_a^2,x)}.$$

2. And introduction of auxiliary PDFs with its own evolution equation $\bar{D}(Q^2,x)$, for which equality between FEV and BEV ditribs. holds *exactly*.

Final elimination of $\bar{D}(Q^2,x)$ provides also precise definition of PDFs in MC factoriz. scheme.

Algebraic validation of NLO-completeness of KrkNLO method

provides again definition of the PDFs in the MC factorization scheme

1. Transform KrkNLO multiparton distributions from BEV to FEV representation (using auxiliary PDFs$\bar{D}$).
2. Integrated and sum over spectator gluons.
3. Expand form-factors and drop all $\mathcal{O}(\alpha_s^2)$ and higher order terms. (Also $\bar{D} \to D$.)
4. Compare resulting formula with that of $\overline{MS}$ in the Catani-Seymour scheme (with $\overline{MS}$ PDFs), verifying that PDFs of KrkNLO are in the MC factorization scheme.

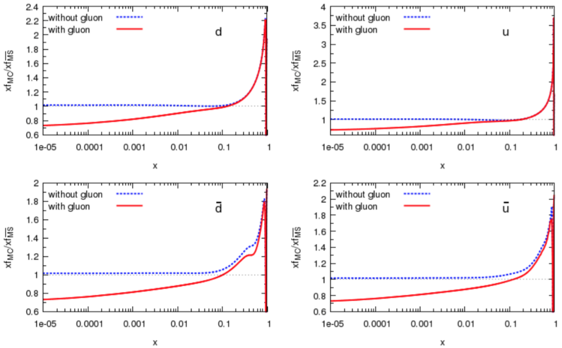
After step 3, with $J = J_{\rm NLO}$ defining any NLO observable, KrkNLO yields:

$$\sigma^{\rm NLO}_{\rm KrkNLO}[J] = \int dx_F\, dx_B\, d\Omega\; (1+\Delta_{VS})\frac{d\sigma}{d\Omega}(s_1,\hat{\theta})J(x_F,x_B,1,0)D^F_{\rm MC}(\hat{s},x_F)D^B_{\rm MC}(\hat{s},x_B)$$
$$+\int dx_F\, dx_B\, d\Omega\Big\{d^5\rho^{\rm NLO}_{q\bar{q}}\; J(x_F,x_B,z_1,k^2_{1T}) - d^5\rho^{\rm LO}_{q\bar{q}}\; J(x_F,x_B,1,0)\Big\}D^F_{\rm MC}(\hat{s},x_F)D^B_{\rm MC}(\hat{s},x_B),$$

The Catani-Seymour scheme analogous fixed order NLO functional $\sigma^{\rm NLO}_{CS}[J]$ is identical, provided $\mu^2 \to \hat{s}$ and $D_{\overline{MS}} \to D_{\rm MC}$, see Appendix B in [arXiv:1503.06849].

PDFs in MC factorization scheme

Ratios of MC PDFs to the standard $\overline{MS}$ PDFs at $Q^2 = 100\mathrm{GeV}$.



In MC scheme quark PDF gets contrinution from gluon! Gluon untouched so far...

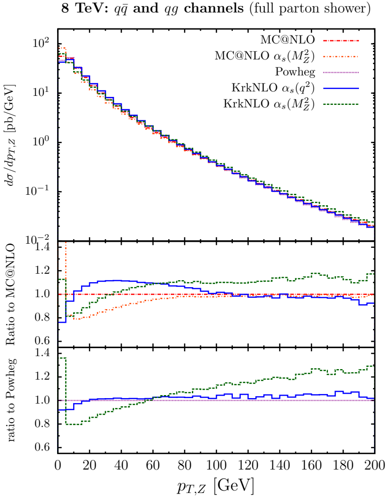
$$f^{\mathrm{MC}}_{q(\bar{q})}(x,Q^2) = f^{\overline{\mathrm{MS}}}_{q(\bar{q})}(x,Q^2) + \int_x^1 \frac{dz}{z} f^{\overline{\mathrm{MS}}}_{q(\bar{q})}\left(\frac{x}{z},Q^2\right)\Delta C_{2q}(z) + \int_x^1 \frac{dz}{z} f^{\overline{\mathrm{MS}}}_{g}\left(\frac{x}{z},Q^2\right)\Delta C_{2g}(z), \qquad f_g^{\mathrm{MC}} = f_g^{\overline{\mathrm{MS}}}.$$

$$\Delta C_{2g}(z) = C^{\overline{\mathrm{MS}}}_{2g}(z) - C^{\mathrm{MC}}_{2g}(z) = \frac{\alpha_s}{2\pi} T_R \left\{ \left[z^2 + (1-z)^2\right] \ln\frac{(1-z)^2}{z} + 2z(1-z) \right\},$$

$$\Delta C_{2q}(z) = \frac{1}{2}\left[C^{\overline{\mathrm{MS}}}_{2q}(z) - C^{\mathrm{MC}}_{2q}(z)\right] = \frac{\alpha_s}{2\pi} C_F \left[\frac{1+z^2}{1-z}\ln\frac{(1-z)^2}{z} + 1 - z\right].$$

Transverse momentum distribution comparisons (main result)

between two versions of KrkNLO and MC@NLO or POWHEG.

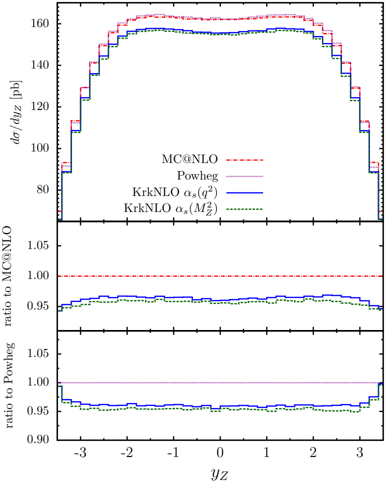


- Differences of order 10-20%. Are they justifiable at NLO level?

Comparison of the rapidity distribution (main result)

between two versions of KrkNLO and MC@NLO or POWHEG.

8 TeV: $q\bar{q}$ **and** qg **channels** (full parton shower)

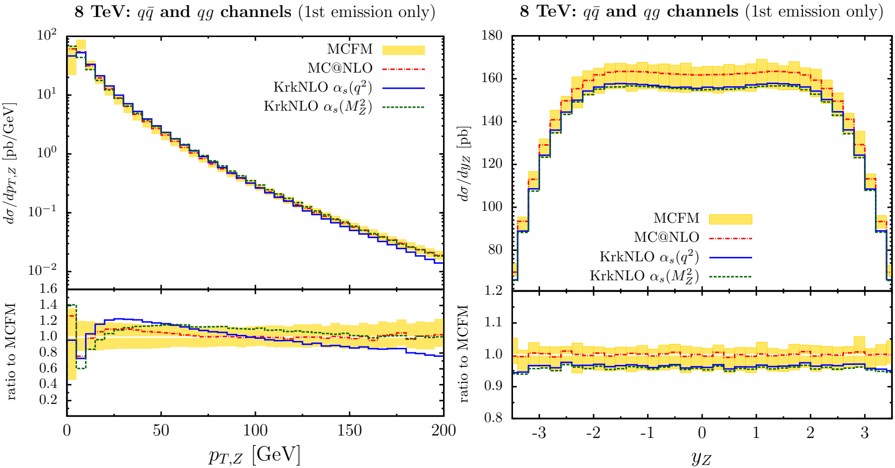


	$\sigma_{\mathrm{tot}}^{q\bar{q}+qg}$ [pb]
MCFM	1086.5 ± 0.1
MC@NLO	1086.5 ± 0.1
POWHEG	1084.2 ± 0.6
KrkNLO $\alpha_s(q^2)$	1045.4 ± 0.1
KrkNLO $\alpha_s(M_Z^2)$	1039.0 ± 0.1

Differences in rapidity distr. normalization the same as in table of total xsect.

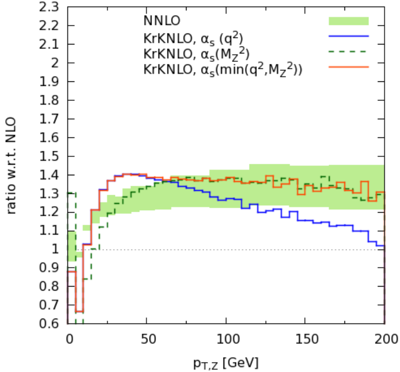
Transverse momentum and rapidity distributions

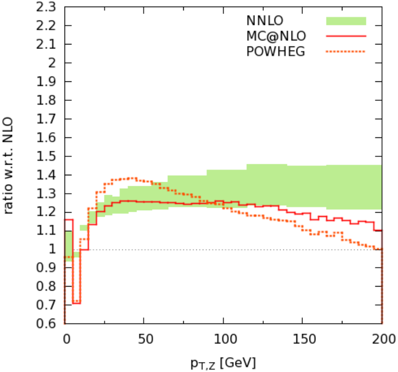
from MCFM, MC@NLO and two versions of KrkNLO.



- Factorization and renormalization scale varied by 2 and 1/2 (independently).
- 10-20% differences well within uncertainty band typical for NLO.

Good agreement of KrkNLO with NNLO fixed order





- The Z-boson transverse-momentum distributions from KrkNLO compared with the fixed-order NNLO result from the DYNNLO (left).
- Similar comparisons for POWHEG and MCatNLO are also shown (right).
- All distributions are divided by the NLO results from MCFM.
- KrkNLO closer to NNLO than POWHEG and MCatNLO.

Outlook

- Short term plan: KrkNLO for Higgs production process, on top of HERWIG++.
- Completing methodology of NLO corrections (in exclusive form) to PS MC shower.
- KrkNLO method for NNLO hard process, i.e. KrkNNLO.

Summary

- **An alternative (to MC@NLO or POWHEG) scenario for NLO-corrected hard proc. + LO PSMC is working well.**
- Parton shower MC implementing complete NLO DGLAP in the ladders in exclusive way is well advanced.
- Long term: N+NLO: NLO ladder + NNLO hard process.
- Most likely application: high quality QCD+EW+QED MC with hard process like $W/Z/H$ boson production at high luminosity LHC.
- Potential gains from new QCD methods are:
  – reducing h.o. QCD uncertainties
  – easier implementation of NLO and NNLO corrections to hard process.
  – better environment for low x resumm. (CCFM),
  – and more...