Higgs boson(s) in LR model

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Mainly based on:
"Lepton Number Violation in Higgs Decay at LHC"
Phys.Rev.Lett. 115 (2015) 081802.

and

"Scalar sector in LR model"
in preparation

with

M. Nemevšek and F. Nesti

I acknowledge G.Senjanovic for useful discussions.

Ustron 17/09/2015

Outline

- Open problem in SM: the origin of neutrino masses.

- The key: A new proper gauge symmetry spontaneously broken. (new Higgs boson) LR extension of the SM.

- Phenomenology: Lepton number violation (LNV). (Majorana neutrino, Keung-Senjanovic process,neutrinoless 2-beta decay…)

Higgs boson in the Standard Model

The Higgs boson (h) discovery is the last triumph of the SM:

- it provides the masses of all charged fermions

- the essence of the Higgs mechanism is that the decay rate of h to two (charged)fermions is $\propto m_f^{\ 2}$

No coupling with neutrino, no decay rate

$$m_\nu = 0 \qquad \longleftrightarrow \qquad \Gamma_{h\to\nu\nu} = 0$$

From SM to a theory of the neutrino mass

Taking care of the main esthetic defect of SM, a complete asymmetry between L & R, a natural theory for neutrino mass emerges:

$$SU(3)_c \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L} \Rightarrow SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$$

[Pati-Salam '74, Mohapatra-Senjanovic '75]

→ via a new Higgs boson

Plus a generalized parity relating left and right: $g_L = g_R$

$$Q_{el} = T_{3L} + T_{3R} + \frac{B-L}{2}$$

$$Q_L = \begin{pmatrix} u \\ d \end{pmatrix}_L \qquad Q_R = \begin{pmatrix} u \\ d \end{pmatrix}_R$$

$$\Psi_L = \begin{pmatrix} \nu_l \\ l \end{pmatrix}_L \qquad \Psi_R = \begin{pmatrix} \nu_l \\ l \end{pmatrix}_R$$

$$Q_L \in (3,2,1,1/3)$$
$$Q_R \in (3,1,2,1/3)$$
$$\Psi_L \in (1,2,1,-1)$$
$$\Psi_R \in (1,1,2,-1)$$

→ A RH neutrino, gauge interacting

RH current ⟶ NP contributions to $0\nu 2\beta$ decay

[Mohapatra,Senjanovic '81]
[Tello,Nemevsek,Nesti,Senjanovic,Vissani 2011]

Higgs Sector

A bi-doublet

$\Phi \in (2_L, 2_R, 0)$

Two triplets

$\Delta_L \in (3_L, 1_R 2)$

$\Delta_R \in (1_L, 3_R, 2)$

[Senjanovic '79]

Vevs

$$\begin{pmatrix} k & 0 \\ 0 & k' e^{i\alpha} \end{pmatrix}$$

$$v_L << k << v_R \qquad v_L \propto 1/v_R$$

$$\tan\beta = k'/k \equiv x < 1$$

Vevs

$$\begin{pmatrix} 0 & 0 \\ v_{L,R} & 0 \end{pmatrix}$$

The potential contains all the possible quadratic and quartic terms in Φ and $\Delta_{L,R}$

A superposition of these field gives the physical (dynamical) ones, the spectrum contains:

Higgs Sector

Mixing the two Higgs bosons

	mass^2	states
h	$\frac{k^2(4\lambda_1\rho_1-\alpha_1^2)}{\rho_1}$	$\frac{1}{\sqrt{2}}(Re\phi_1 + xRe\phi_2 - \frac{k\alpha_1}{2\rho_1 v_R} Re\delta_R)$
H	$v_R^2\left(\frac{4(4\alpha_2^2+(2\lambda_2+\lambda_3)(\alpha_3-4\rho_1))k^2}{v_R^2(\alpha_3-4\rho_1)}+\alpha_3\right)$	$\frac{1}{\sqrt{2}}(Re\phi_2 - xRe\phi_1 + \frac{4k\alpha_2}{v_R(\alpha_3-4\rho_1)}Re\delta_R)$
H’	$9(\lambda_3-2\lambda_2)k^2+v_R^2\alpha_3$	$\frac{1}{\sqrt{2}}(Im\phi_2+xIm\phi_1)$
Δ_R	$\frac{(\alpha_1^2(\alpha_3-4\rho_1)-16\alpha_2^2\rho_1)k^2}{(\alpha_3-4\rho_1)\rho_1}+4v_R^2\rho_1$	$\frac{1}{\sqrt{2}}(Re\delta_R+\frac{k\alpha_1}{2\rho_1 v_R}Re\phi_1-\frac{4k\alpha_2}{v_R(\alpha_3-4\rho_1)}Re\phi_2)$
Δ_L	$v_R^2(\rho_3-2\rho_1)$	$\frac{1}{\sqrt{2}}Re\delta_L$
Δ'_L	$v_R^2(\rho_3-2\rho_1)$	$\frac{1}{\sqrt{2}}Im\delta_L$
H^-	$\frac{1}{2}\left(k^2+2v_R^2\right)\alpha_3$	$\frac{1}{\sqrt{2}}(\phi_2^- + x\phi_1^- + \frac{k}{\sqrt{2}v_R}\delta_R^-)$
Δ_R^{--}	$\alpha_3k^2+4v_R^2\rho_2$	$\frac{1}{\sqrt{2}}\delta_R^{--}$
Δ_L^-	$\frac{\alpha_3k^2}{2}+v_R^2(\rho_3-2\rho_1)$	$\frac{1}{\sqrt{2}}\delta_L^-$
Δ_L^{--}	$\alpha_3k^2+v_R^2(\rho_3-2\rho_1)$	$\frac{1}{\sqrt{2}}\delta_L^{--}$

Higgs Bosons

FC (pseudo)-scalar

[Senjanovic ’79]
[Gunion,Kayser, Olness’ 89]
[Duka, Gluza, Zralek 2000]
[Kiers,Assis,Petrov 2005]
[Zhang,An,Ji,Mohapatra 2007]
[A.M.,Nemevsek,Nesti in prep.]

Mixing the two Higgs bosons

$$\theta \equiv \frac{\alpha_1 k}{2\rho_1 v_R} < 40\%$$ 2-sigma C.L.

[Falkowski,Gross,Lebedev 2015]

Probing neutrino masses

The new Higgs boson $m_{\Delta_R}{}^2 \approx 4\rho_1 v_R{}^2$

Majorana terms $L_{yuk} = (y_\Delta \overline{\psi}_R \psi_R{}^c \Delta_R + R \leftrightarrow L) + h.c.$

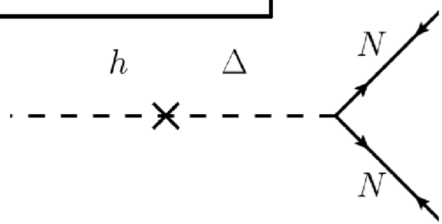
$$m_N = 2 y_\Delta v_R \qquad M_{W_R} = g v_R \qquad m_\nu = -m_D{}^T m_N^{-1} m_D$$

See-saw

[Minkowski '77, Mohapatra Senjanovic '79,
Glashow '79; Yanagida '79]

$$\Gamma_{\Delta \to NN} \propto y_\Delta{}^2$$

Via the mixing

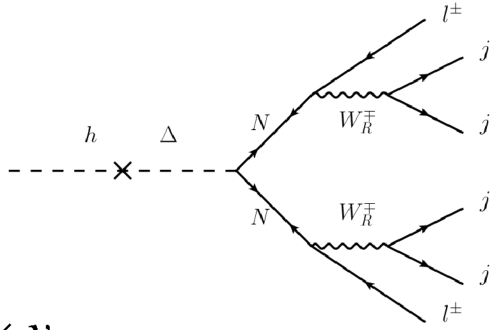


$$\frac{\Gamma_{NN}}{\Gamma_{b\bar{b}}} \simeq \frac{\tan\theta^2}{3}\left(\frac{m_N}{m_b}\right)^2\left(\frac{M_W}{M_{W_R}}\right)^2\left(1-\frac{4m_N^2}{m_h^2}\right)^{\frac{3}{2}} \qquad (c\,\tau_N^0)^{-1} \simeq \frac{G_F^2 m_N^5}{16\pi^3}\left(\frac{M_W}{M_{W_R}}\right)^4$$

[A.M.,Nemevsek,Nesti 2015] [Nemevsek,Nesti,Senjanovic,Zhang 2011]

Probing neutrino masses

The SM-like Higgs boson



$$\frac{\Gamma_{NN}}{\Gamma_{b\bar{b}}} \simeq \frac{\tan\theta^2}{3}\left(\frac{m_N}{m_b}\right)^2\left(\frac{M_W}{M_{W_R}}\right)^2\left(1-\frac{4m_N^2}{m_h^2}\right)^{\frac{3}{2}} \rightarrow \theta \times y_\Delta$$

$$(c\,\tau_N^0)^{-1} \simeq \frac{G_F^2 m_N^5}{16\pi^3}\left(\frac{M_W}{M_{W_R}}\right)^4 \longrightarrow M_{W_R}$$

(displacement of N decay products)

[A.M.,Nemevsek, Nesti 2015]

Invariant mass $\longrightarrow$ m_N

Global fit on Higgs data $\longrightarrow$ θ

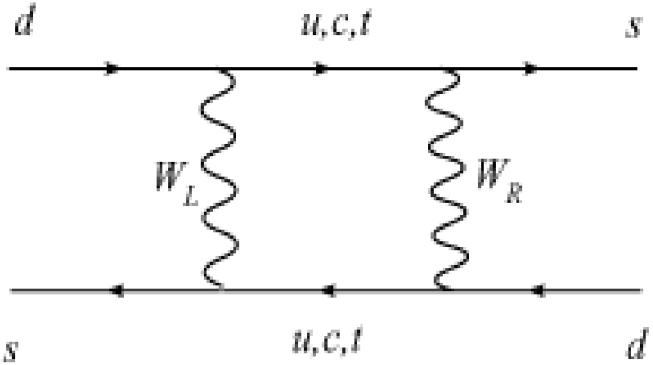
To complete the understanding of neutrino mass origin, one would clearly like to observe Δ_R and the associated gauge boson that provides the gauge symmetry protection.
Ideally KS⟶ M_{W_R},M_N⟶ predict Y_D, then M_N decay

[Nemevsek,Senjanovic, Tello PRL 2012]

➡

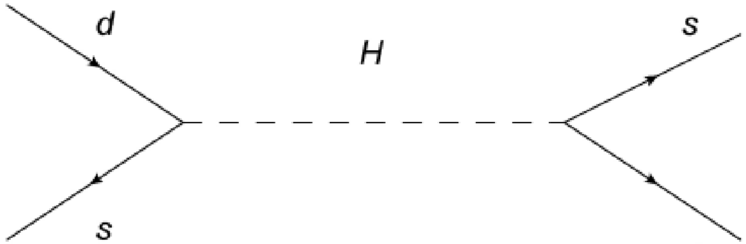
Theoretical limits on the model

Theoretical constrains



New box diagram from charged gauge interactions. V_L and V_R entering.

[Beall, Bander ,Soni ‘82, Ecker,Grimus ’85]



$$L \propto \overline{d}_L V_L^+ m_u V_R d_R H$$

Neutral Heavy Higgs flavor Changing at tree level.
Same V_L and V_R structure.

[G. Senjanovic, P. Senjanovic ’80]

Predictivity of the model

Analytic solution for V_R

[Senjanovic, Tello PRL 2014]

In the leptonic sector is possible to determine the Yukawa coupling from the neutrino masses and mixing.

[Nemevsek,Senjanovic, Tello 2012]

Theoretical constrains

Meson oscillations

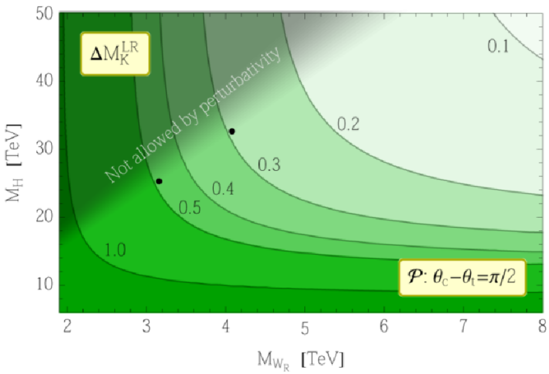


FIG. 9. Correlated bounds on M_R and M_{W_R} (region above the curves) for $|\Delta M_K^{LR}|/\Delta M_K^{exp} < 1.0, ..., 0.1$ and for $\theta_c - \theta_t = \pi/2$ in the case of $\mathcal{P}$ parity.

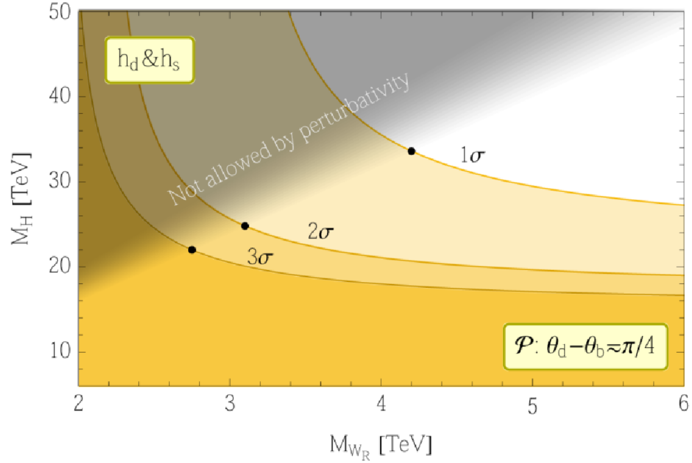


FIG. 10. Combined constraints on M_R and M_{W_R} from ε, ε' B_d and B_s mixings obtained in the $\mathcal{P}$ parity case from the numerical fit of the Yukawa sector of the model.

[A.M.,Nemevsek,Nesti,Senjanovic 2010]
[Bertolini, A.M.,Nesti ,2014]

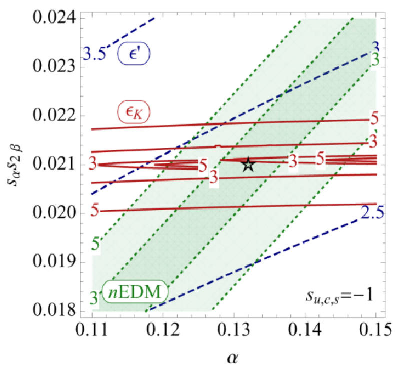


FIG. 3. Combined CPV constraints in the LRSM-$\mathcal{P}$ extended with an "invisible" axion. The solution for V_R obtained from (16) with $s_{u,c,s} = -1$ and all others $+1$. Contours in dashed red, solid blue and dotted green show a bound on M_{W_R} in TeV units coming from $\varepsilon', \varepsilon_K$ and nEDM via $\bar{\theta}_{\text{ind}}$, respectively. The star denotes a point where all constraints are satisfied and $M_{W_R} \gtrsim 3$ TeV.

nEDM (without strong CP) [A.M.,Nemevsek 2014]
and
epsilon, epsilon-prime

[Bertolini,Eeg, A.M.,Nesti 2013
[Bertolini, A.M.,Nesti 2014]

Theoretical constrains

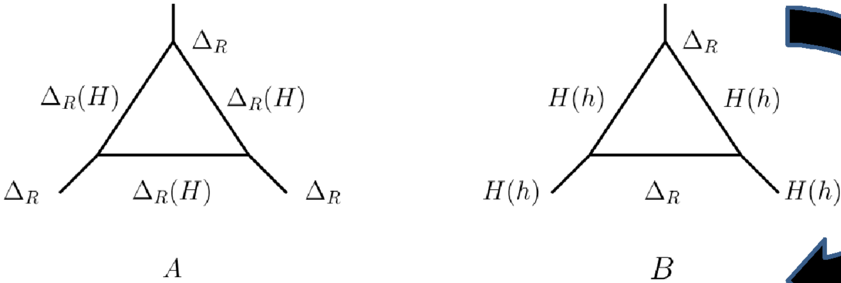
	mass^2	states
h	$\frac{k^2(4\lambda_1\rho_1-\alpha_1^2)}{\rho_1}$	$\frac{1}{\sqrt{2}}(Re\phi_1 + xRe\phi_2 - \frac{k\alpha_1}{2\rho_1 v_R}Re\delta_R)$
H	$v_R^2\left(\frac{4(4\alpha_2^2+(2\lambda_2+\lambda_3)(\alpha_3-4\rho_1))k^2}{v_R^2(\alpha_3-4\rho_1)}+\alpha_3\right)$	$\frac{1}{\sqrt{2}}(Re\phi_2 - xRe\phi_1 + \frac{4k\alpha_2}{v_R(\alpha_3-4\rho_1)}Re\delta_R)$
H’	$9(\lambda_3-2\lambda_2)k^2+v_R^2\alpha_3$	$\frac{1}{\sqrt{2}}(Im\phi_2 + xIm\phi_1)$
Δ_R	$\frac{(\alpha_1^2(\alpha_3-4\rho_1)-16\alpha_2^2\rho_1)k^2}{(\alpha_3-4\rho_1)\rho_1}+4v_R^2\rho_1$	$\frac{1}{\sqrt{2}}(Re\delta_R + \frac{k\alpha_1}{2\rho_1 v_R}Re\phi_1 - \frac{4k\alpha_2}{v_R(\alpha_3-4\rho_1)}Re\phi_2)$
Δ_L	$v_R^2(\rho_3-2\rho_1)$	$\frac{1}{\sqrt{2}}Re\delta_L$
Δ_L'	$v_R^2(\rho_3-2\rho_1)$	$\frac{1}{\sqrt{2}}Im\delta_L$
H^-	$\frac{1}{2}(k^2+2v_R^2)\alpha_3$	$\frac{1}{\sqrt{2}}(\phi_2^- + x\phi_1^- + \frac{k}{\sqrt{2}v_R}\delta_R^-)$
Δ_R^{--}	$\alpha_3 k^2+4v_R^2\rho_2$	$\frac{1}{\sqrt{2}}\delta_R^{--}$
Δ_L^-	$\frac{\alpha_3 k^2}{2}+v_R^2(\rho_3-2\rho_1)$	$\frac{1}{\sqrt{2}}\delta_L^-$
Δ_L^{--}	$\alpha_3 k^2+v_R^2(\rho_3-2\rho_1)$	$\frac{1}{\sqrt{2}}\delta_L^{--}$

α_3 has to be large for a low scale theory

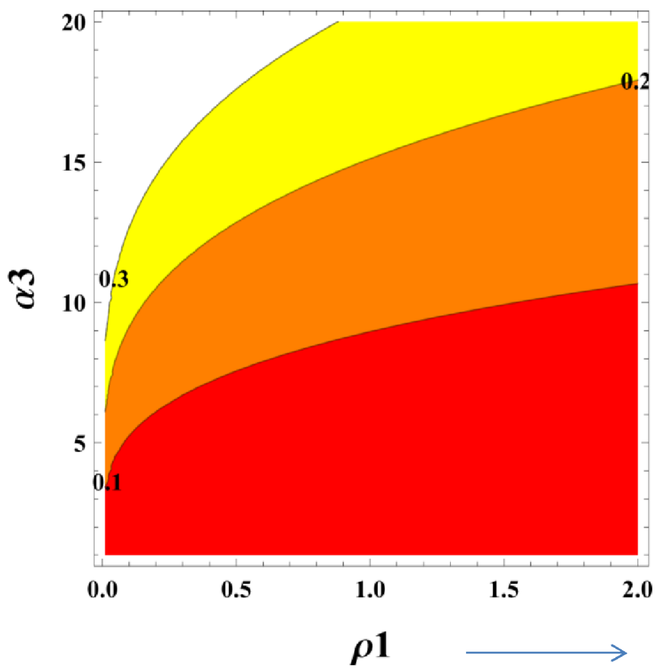
How large can α_3 be to keep the theory in a good perturbative regime in the scalar sector processes?

Theoretical constrains

Perturbativity

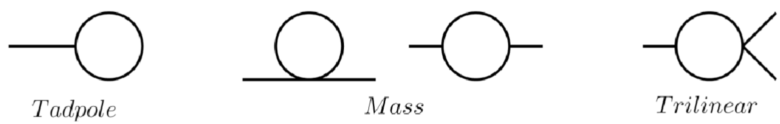


Matched with tree-level equivalent



→ Related to the new Higgs mass

After renormalization of the divergent contributions:

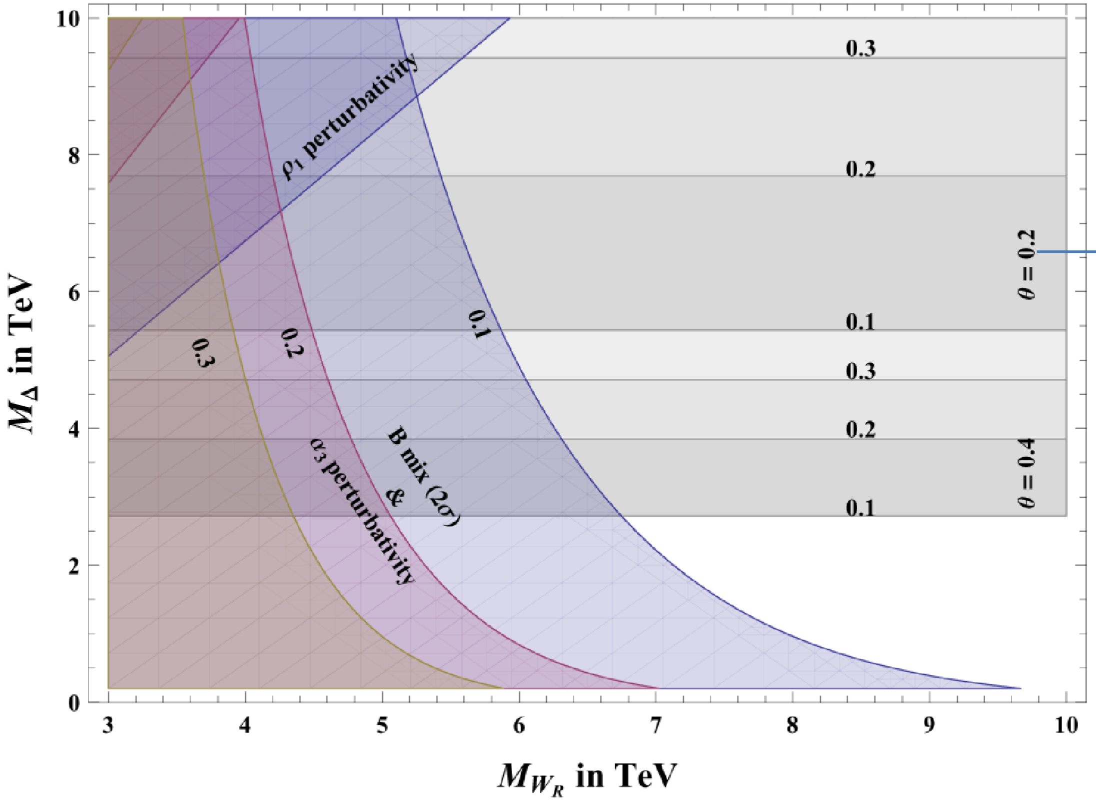


[A.M.,Nemevsek ,Nesti in preparation]

Theoretical constrains

All together

[A.M.,Nemevsek ,Nesti in preparation]



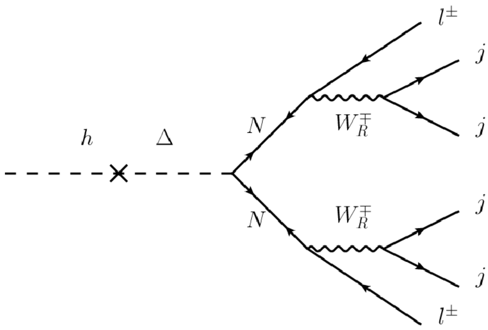
Large mixing even with a slightly heavy Delta

$$\theta \equiv \frac{\alpha_1 k}{2\rho_1 v_R}$$

In a pessimistic scenario the mixing could be the only way to probe neutrino masses at LHC.

LNV Higgs decay at LHC

Same sign muons: **h⟶$\mu\mu$ + jets**



- **Signal vs SM background**: same sign muons vs *WZ+ZZ+WW2j+ttbar (simulated), QCD (estimated as x2.5)*
- **Collider simulation**: Madgraph5 (event generator) + Pythia6(hadronization) + Delphes3(detector)

[A.M.,Nemevsek,Nesti 2015]

Process	No cuts	Imposed cuts				
		$\mu^\pm\mu^\pm + n_j$	$\not{E}_T$	p_T	m_T	m_{inv}
WZ	2 M	544	143	78	40	20
ZZ	1 M	55	29	16	12	8
$W^\pm W^\pm 2j$	389	115	16	5	3	1
$t\bar{t}$	10 M	509	97	40	22	14
Signal (40)	543	44	43	41	38	37

TABLE I. Number of expected events at the 13 TeV LHC run with $\mathcal{L} = 100\,\text{fb}^{-1}$ after cuts described in the text. The signal is generated with 40 GeV, $\sin\theta = 10\,\%$, $M_{W_R} = 3\,\text{TeV}$ and $n_j = 1, 2, 3$.

See the talk by M. Nemevsek

Model-file available to: https://sites.google.com/site/leftrighthep/

Modified from the version in: [Roitgrund,Eilam,Bar-Shalom 2014]

LNV Higgs decay at LHC

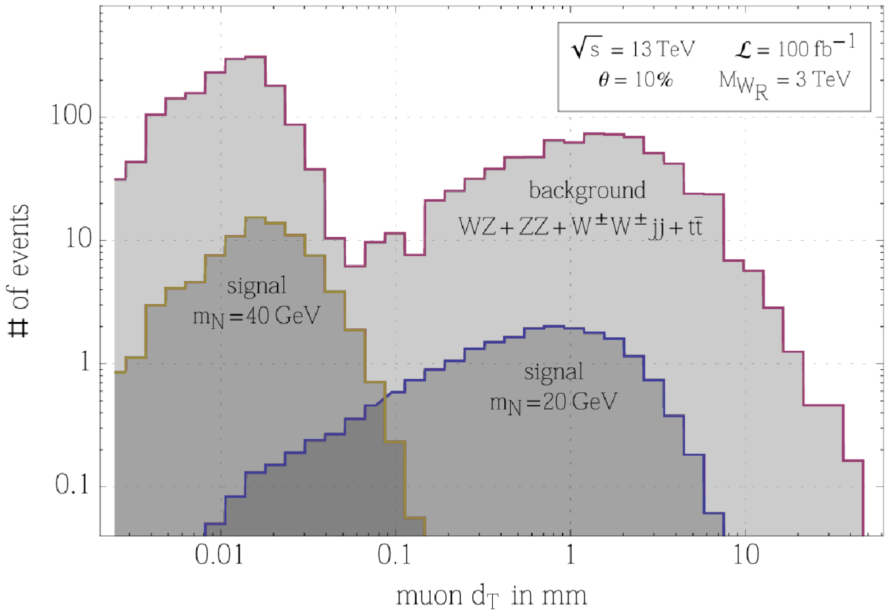
Taking advantage of displaced vertex.

- Muons are both displaced: *N lifetime depending on m_N and M_{W_R}*

$$(c\,\tau_N^0)^{-1} \simeq \frac{G_F^2 m_N^5}{16\pi^3}\left(\frac{M_W}{M_{W_R}}\right)^4$$

- We require two displacements and employ a sliding window cut:

$$L/10 < d_T < 5 \times L$$



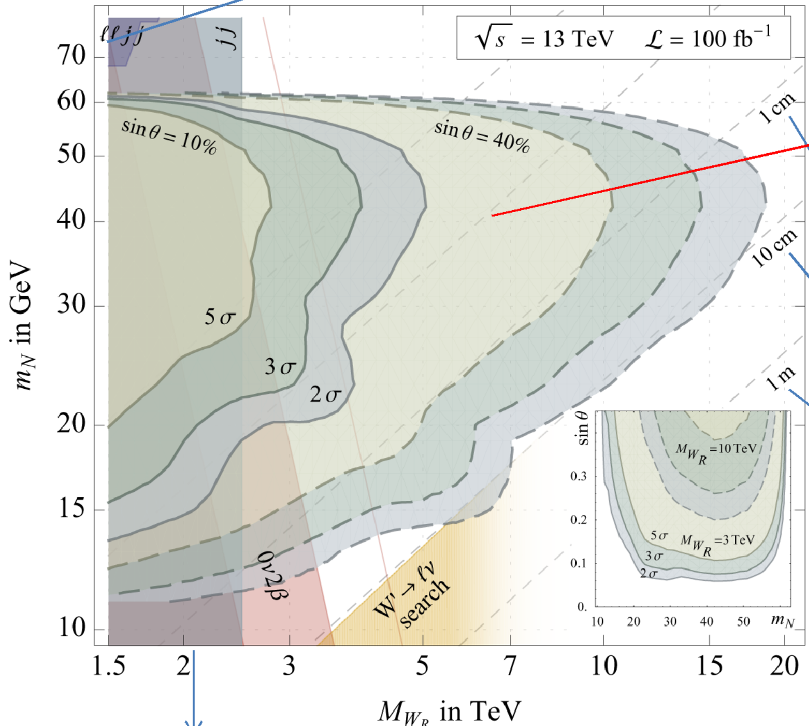
[A.M.,Nemevsek,Nesti 2015]

LNV Higgs decay at LHC

LHC sensitivity

Keung-Senjanovic process lljj

[Keung,Senjanovic 83]



LNV Higgs decay: $h \longrightarrow \mu\mu$ + jets

N lifetime

W' dijet search

[A.M.,Nemevsek,Nesti 2015]

Outlook

- Left-Right model as a complete theory of the neutrino masses.
- Implications of a low scale left-right symmetry: low energy process and perturbativity.
- Despite several constraints on the theory(=high predictivity), the SM-like Higgs boson within the model could serve as gateway on NP via LNV.
- possibility to probe parity restoration up to 20 TeV through LNV Higgs decay.

Thanks!

The choice of Left-Right symmetry is not univocal

$$\mathcal{P}: \begin{cases} Q_L \leftrightarrow Q_R \\ \Phi \rightarrow \Phi^\dagger \end{cases} \qquad \mathcal{C}: \begin{cases} Q_L \leftrightarrow (Q_R)^c \\ \Phi \rightarrow \Phi^T \end{cases}$$

Which lead respectively to

$$\mathcal{P}: \; Y = Y^\dagger, \qquad \mathcal{C}: \; Y = Y^T$$

[A.M., Nemevsek,Nesti,Senjanovic, 2010]

- The case of "**P**" is the original one, hence it is the most known in literature. It can be interesting for nEDM.

- The case of "**C**" should be considered equally. It is also interesting in SO(10) GUT scenario, where charge conjugation enters automatically in the algebra. (For instance the fermions and their charge conjugated in the same important representation **16**F).

The potential with P symmetry

$$
\begin{aligned}
\mathcal{V} = & -\mu_1^2 \mathrm{Tr}[\phi^\dagger \phi] - \mu_2^2 \left(\mathrm{Tr}\left[\tilde{\phi}\phi^\dagger\right] + \mathrm{Tr}\left[\tilde{\phi}^\dagger \phi\right]\right) - \mu_3^2 \left(\mathrm{Tr}\left[\Delta_L \Delta_L^\dagger\right] + \mathrm{Tr}\left[\Delta_R . \Delta_R^\dagger\right]\right) \\
& + \lambda_1 (\mathrm{Tr}[\phi^\dagger \phi])^2 + \lambda_2 \left(\left(\mathrm{Tr}\left[\tilde{\phi}\phi^\dagger\right]\right)^2 + \left(\mathrm{Tr}\left[\tilde{\phi}^\dagger \phi\right]\right)^2 \right) + \lambda_3 \mathrm{Tr}\left[\tilde{\phi}\phi^\dagger\right] \mathrm{Tr}\left[\tilde{\phi}^\dagger \phi\right] \\
& + \lambda_4 \mathrm{Tr}[\phi^\dagger \phi] \left(\mathrm{Tr}\left[\tilde{\phi}\phi^\dagger\right] + \mathrm{Tr}\left[\tilde{\phi}^\dagger \phi\right]\right) + \rho_1 \left(\left(\mathrm{Tr}\left[\Delta_L . \Delta_L^\dagger\right]\right)^2 + \left(\mathrm{Tr}\left[\Delta_R . \Delta_R^\dagger\right]\right)^2 \right) \\
& + \rho_2 \left(\mathrm{Tr}\left[\Delta_L \Delta_L\right] \mathrm{Tr}\left[\Delta_L^\dagger \Delta_L^\dagger\right] + \mathrm{Tr}\left[\Delta_R \Delta_R\right] \mathrm{Tr}\left[\Delta_R^\dagger \Delta_R^\dagger\right]\right) \\
& + \rho_3 \mathrm{Tr}\left[\Delta_L \Delta_L^\dagger\right] \mathrm{Tr}\left[\Delta_R \Delta_R^\dagger\right] + \rho_4 \left(\mathrm{Tr}\left[\Delta_L \Delta_L\right] \mathrm{Tr}\left[\Delta_R^\dagger \Delta_R^\dagger\right] \right. \\
& \left. + \mathrm{Tr}\left[\Delta_L^\dagger \Delta_L^\dagger\right] \mathrm{Tr}\left[\Delta_R \Delta_R\right]\right) + \alpha_1 \mathrm{Tr}[\phi^\dagger \phi] \left(\mathrm{Tr}\left[\Delta_L \Delta_L^\dagger\right] + \mathrm{Tr}\left[\Delta_R \Delta_R^\dagger\right]\right) \\
& + \alpha_2 e^{i\delta_2} \left(\mathrm{Tr}\left[\tilde{\phi}\phi^\dagger\right] \mathrm{Tr}\left[\Delta_L \Delta_L^\dagger\right] + \mathrm{Tr}\left[\tilde{\phi}^\dagger \phi\right] \mathrm{Tr}\left[\Delta_R \Delta_R^\dagger\right]\right) \\
& + \alpha_2 e^{-i\delta_2} \left(\mathrm{Tr}\left[\phi\tilde{\phi}^\dagger\right] \mathrm{Tr}\left[\Delta_L \Delta_L^\dagger\right] + \mathrm{Tr}\left[\phi^\dagger \tilde{\phi}\right] \mathrm{Tr}\left[\Delta_R \Delta_R^\dagger\right]\right) \\
& + \alpha_3 \left(\mathrm{Tr}\left[\phi\phi^\dagger \Delta_L . \Delta_L^\dagger\right] + \mathrm{Tr}\left[\phi^\dagger \phi \Delta_R \Delta_R^\dagger\right]\right) \\
& + \beta_1 \left(\mathrm{Tr}\left[\phi \Delta_R \phi^\dagger \Delta_L^\dagger\right] + \mathrm{Tr}\left[\phi^\dagger \Delta_L \phi \Delta_R^\dagger\right]\right) \\
& + \beta_2 \left(\mathrm{Tr}\left[\tilde{\phi} \Delta_R \phi^\dagger \Delta_L^\dagger\right] + \mathrm{Tr}\left[\tilde{\phi}^\dagger \Delta_L \phi \Delta_R^\dagger\right]\right) \\
& + \beta_3 \left(\mathrm{Tr}\left[\phi \Delta_R \tilde{\phi}^\dagger \Delta_L^\dagger\right] + \mathrm{Tr}\left[\phi^\dagger \Delta_L \tilde{\phi} \Delta_R^\dagger\right]\right)
\end{aligned}
$$

The potential with C symmetry

$$
\begin{aligned}
\mathcal{V} = & -\mu_1^2 \mathrm{Tr}[\phi^\dagger \phi] - \mu_2^2 \left(e^{i\delta_{\mu_2}} \mathrm{Tr}\left[\tilde{\phi}\phi^\dagger\right] + h.c. \right) - \mu_3^2 \left(\mathrm{Tr}\left[\Delta_L \Delta_L^\dagger\right] + \mathrm{Tr}\left[\Delta_R \Delta_R^\dagger\right] \right) \\
& + \lambda_1 (\mathrm{Tr}[\phi^\dagger \phi])^2 + \lambda_2 \left(e^{i\delta_{\lambda_2}} \left(\mathrm{Tr}\left[\tilde{\phi}\phi^\dagger\right] \right)^2 + h.c. \right) + \lambda_3 \mathrm{Tr}\left[\tilde{\phi}\phi^\dagger\right] \mathrm{Tr}\left[\tilde{\phi}^\dagger \phi\right] \\
& + \lambda_4 \mathrm{Tr}[\phi^\dagger \phi] \left(e^{i\delta_{\lambda_4}} \mathrm{Tr}\left[\tilde{\phi}\phi^\dagger\right] + h.c. \right) + \rho_1 \left(\left(\mathrm{Tr}\left[\Delta_L \Delta_L^\dagger\right] \right)^2 + \left(\mathrm{Tr}\left[\Delta_R \Delta_R^\dagger\right] \right)^2 \right) \\
& + \rho_2 \left(\mathrm{Tr}\left[\Delta_L \Delta_L\right] \mathrm{Tr}\left[\Delta_L^\dagger \Delta_L^\dagger\right] + \mathrm{Tr}\left[\Delta_R \Delta_R\right] \mathrm{Tr}\left[\Delta_R^\dagger \Delta_R^\dagger\right] \right) \\
& + \rho_3 \mathrm{Tr}\left[\Delta_L \Delta_L^\dagger\right] \mathrm{Tr}\left[\Delta_R \Delta_R^\dagger\right] + \rho_4 \left(e^{i\delta_{\rho_4}} \mathrm{Tr}\left[\Delta_L \Delta_L\right] \mathrm{Tr}\left[\Delta_R^\dagger \Delta_R^\dagger\right] + h.c. \right) \\
& + \alpha_1 \mathrm{Tr}[\phi^\dagger \phi] \left(\mathrm{Tr}\left[\Delta_L \Delta_L^\dagger\right] + \mathrm{Tr}\left[\Delta_R \Delta_R^\dagger\right] \right) \\
& + \alpha_2 e^{i\delta_2} \left(\mathrm{Tr}\left[\tilde{\phi}\phi^\dagger\right] \mathrm{Tr}\left[\Delta_L \Delta_L^\dagger\right] + \mathrm{Tr}\left[\tilde{\phi}^\dagger \phi\right] \mathrm{Tr}\left[\Delta_R \Delta_R^\dagger\right] \right) \\
& + \alpha_2 e^{-i\delta_2} \left(\mathrm{Tr}\left[\phi\tilde{\phi}^\dagger\right] \mathrm{Tr}\left[\Delta_L \Delta_L^\dagger\right] + \mathrm{Tr}\left[\phi^\dagger \tilde{\phi}\right] \mathrm{Tr}\left[\Delta_R \Delta_R^\dagger\right] \right) \\
& + \alpha_3 \left(\mathrm{Tr}\left[\phi\phi^\dagger \Delta_L \Delta_L^\dagger\right] + \mathrm{Tr}\left[\phi^\dagger \phi \Delta_R \Delta_R^\dagger\right] \right) \\
& + \beta_1 \left(e^{i\delta_{\beta_1}} \mathrm{Tr}\left[\phi \Delta_R \phi^\dagger \Delta_L^\dagger\right] + h.c. \right) \\
& + \beta_2 \left(e^{i\delta_{\beta_2}} \mathrm{Tr}\left[\tilde{\phi} \Delta_R \phi^\dagger \Delta_L^\dagger\right] + h.c. \right) \\
& + \beta_3 \left(e^{i\delta_{\beta_3}} \mathrm{Tr}\left[\phi \Delta_R \tilde{\phi}^\dagger \Delta_L^\dagger\right] + h.c. \right)
\end{aligned}
$$

See-saw between the VEVs

$$v_L = \frac{k^2\left(\beta_2\cos(\theta_L) + \beta_3 x^2\cos(2\alpha - \theta_L)\right.}{v_R(2\rho_1 - \rho_3)} + \frac{\left.\beta_1 x\cos(\alpha - \theta_L)\right)}{v_R(2\rho_1 - \rho_3)}$$