Hadronic Contributions to $(g-2)_\mu$ and $\alpha_{\rm QED}(M_Z^2)$

Daisuke Nomura (YITP, Kyoto U.)

talk at 'Matter to the Deepest' at Ustroń, Poland
16 September 2015

Partially based on:

- K. Hagiwara, A. Keshavarzi, A. D. Martin, DN & T. Teubner, work in progress
- K. Hagiwara, R. Liao, A. D. Martin, DN & T. Teubner **(HLMNT)**, J. Phys. **G38** (2011) 085003

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Muon $g-2$: introduction

Lepton magnetic moment $\vec{\mu}$:

$$\boxed{\vec{\mu} = -g\frac{e}{2m}\vec{s}}, \qquad \left(\vec{s} = \frac{1}{2}\vec{\sigma}\ \text{(spin)}, \qquad g = 2 + 2F_2(0)\right)$$

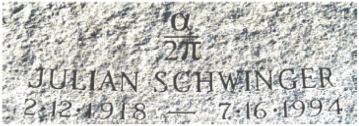
where

$$\overline{u}(p+q)\Gamma^{\mu}u(p) = \overline{u}(p+q)\left(\gamma^{\mu}F_1(q^2) + \frac{i\sigma^{\mu\nu}q_{\nu}}{2m}F_2(q^2)\right)u(p)$$

Anomalous magnetic moment: $a \equiv (g-2)/2$ $(= F_2(0))$

Historically,

- ★ $g = 2$ (tree level, Dirac)
- ★ $a = \alpha/(2\pi)$ (1-loop QED, Schwinger)



Today, still important, since...

★ One of the **most precisely measured** quantities:

$$\boxed{a_{\mu}^{\text{exp}} = 11\ 659\ 208.9(6.3) \times 10^{-10}} \quad [0.5\text{ppm}] \qquad \text{(Bennett } et\ al\text{)}$$

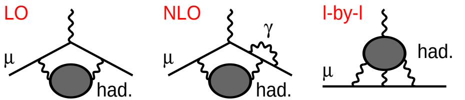
★ **Extremely useful** in **probing/constraining physics beyond the SM**

Introduction: Standard Model prediction for muon $g-2$

QED contribution	11 658 471.808 (0.015)	Kinoshita & Nio, Aoyama et al
EW contribution	15.4 (0.2)	Czarnecki et al
Hadronic contributions		
LO hadronic	**694.9 (4.3)**	HLMNT11
NLO hadronic	−9.8 (0.1)	HLMNT11
light-by-light	10.5 (2.6)	Prades, de Rafael & Vainshtein
Theory TOTAL	**11 659 182.8 (4.9)**	
Experiment	**11 659 208.9 (6.3)**	world avg
Exp − Theory	**26.1 (8.0)**	**3.3 σ discrepancy**

(in units of 10^{-10}. Numbers taken from HLMNT11, arXiv:1105.3149)

n.b.: hadronic contributions:



Improvements in the past few years (1)

- **QED contribution**

  5-loop calculation completed:

  now $a_\mu(\text{QED}) = 11\ 658\ 471.895(08) \times 10^{-10}$

  was $11\ 658\ 471.808(15) \times 10^{-10}$

  (numbers are from Aoyama et al (2012, 2007))

- **EW contribution**

  Higgs-boson mass just fixed:

  now $a_\mu(\text{EW}) = 15.4(1) \times 10^{-10}$ Gnendiger et al '13

  was $15.4(2) \times 10^{-10}$ Czarnecki et al '02

cf: $a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = \begin{cases} (26.1 \pm 8.0) \times 10^{-10} & \text{HLMNT11} \\ (28.7 \pm 8.0) \times 10^{-10} & \text{Davier et al '10} \end{cases}$

Improvements in the past few years (2)

- **NNLO hadronic contribution**

$$a_\mu(\text{had, NNLO}) = (1.24 \pm 0.01) \times 10^{-10}$$

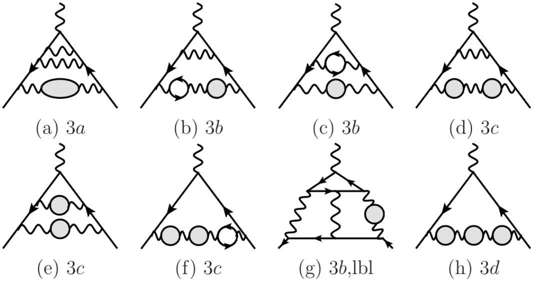


Figure 2: Sample NNLO Feynman diagrams contributing to $a_\mu^{\rm had}$.

Numbers and Figs. from
A. Kurz et al, arXiv:1403.6400

3a:	$+0.80 \times 10^{-10}$
3b:	-0.41×10^{-10}
3b,lbl:	$+0.91 \times 10^{-10}$
3c:	-0.06×10^{-10}
3d:	$+0.0005 \times 10^{-10}$

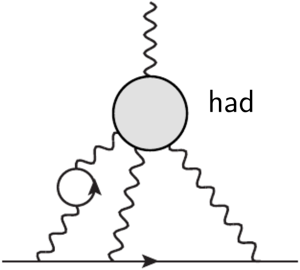
cf: $a_\mu^{\rm exp} - a_\mu^{\rm SM} = \begin{cases} (26.1 \pm 8.0) \times 10^{-10} & \text{HLMNT11} \\ (28.7 \pm 8.0) \times 10^{-10} & \text{Davier et al '10} \end{cases}$

$$a_\mu(\text{had, NLO}) = (-9.8 \pm 0.1) \times 10^{-10} \quad \text{HLMNT11}$$

Improvements in the past few years (3)

- **lbyl-NLO hadronic contribution**

$$a_\mu(\text{had, lbyl-NLO}) = (0.3 \pm 0.2) \times 10^{-10}$$



Number and Fig. from Colangelo et al, arXiv:1403.7512

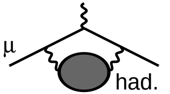
$\Longrightarrow$ **Negligible**. But it is always good to confirm that higher order terms are really negligible.

cf: $$a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = \begin{cases} (26.1 \pm 8.0) \times 10^{-10} & \text{HLMNT11} \\ (28.7 \pm 8.0) \times 10^{-10} & \text{Davier et al '10} \end{cases}$$

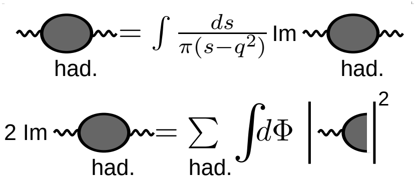
$$a_\mu^{\text{had,lbyl}} = \begin{cases} (10.5 \pm 2.6) \times 10^{-10} & \text{Prades et al, '09} \\ (11.6 \pm 3.9) \times 10^{-10} & \text{Jegerlehner+Nyffeler, '09} \end{cases}$$

Introduction for $a_\mu^{\rm had,LO}$

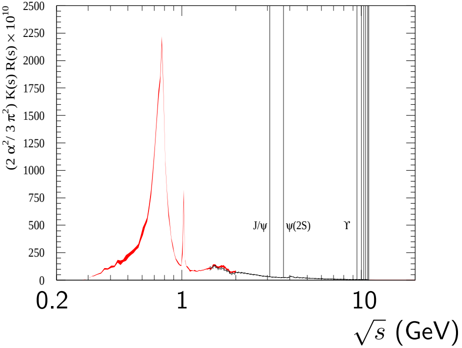
The diagram to be evaluated:



pQCD not useful. Use the dispersion relation and the optical theorem.



$$a_\mu^{\rm had,LO} = \frac{m_\mu^2}{12\pi^3}\int_{s_{\rm th}}^{\infty} ds\ \frac{1}{s}\hat{K}(s)\sigma_{\rm had}(s)$$



- Weight function $\hat{K}(s)/s = \mathcal{O}(1)/s$

$\Longrightarrow$ Lower energies more important

$\Longrightarrow$ $\pi^+\pi^-$ channel: 73% of total $a_\mu^{\rm had,LO}$

Included Hadronic Final States

Channel	Experiments with References
$\pi^+\pi^-$	OLYA [16, 17, 18], OLYA-TOF [19], NA7 [20], OLYA and CMD [21, 22], DM1 [23], DM2 [24], BCF [25, 26], MEA [27, 28], ORSAY-ACO [29], CMD-2 [10, 11, 30]
$\pi^0\gamma$	SND [31, 32]
$\eta\gamma$	SND [32, 33], CMD-2 [34, 35, 36]
$\pi^+\pi^-\pi^0$	ND [22], DM1 [37], DM2 [38], CMD-2 [10, 13, 34, 39], SND [40, 41], CMD [42]
K^+K^-	MEA [27], OLYA [43], BCF [26], DM1 [44], DM2 [45, 46], CMD [22], CMD-2 [34], SND [47]
$K^0_SK^0_L$	DM1 [48], CMD-2 [10, 14, 49], SND [47]
$\pi^+\pi^-\pi^0\pi^0$	M3N [50], DM2 [51], OLYA [52], CMD-2 [53], SND [54], ORSAY-ACO [55], $\gamma\gamma2$ [56], MEA [57]
$\omega(\to\pi^0\gamma)\pi^0$	ND and ARGUS [22], DM2 [51], CMD-2 [53, 58], SND [59, 60], ND [61]
$\pi^+\pi^-\pi^+\pi^-$	ND [22], M3N [50], CMD [62], DM1 [63, 64], DM2 [51], OLYA [65], $\gamma\gamma2$ [66], CMD-2 [53, 67, 68], SND [54], ORSAY-ACO [55]
$\pi^+\pi^-\pi^+\pi^-\pi^0$	MEA [57], M3N [50], CMD [22, 62], $\gamma\gamma2$ [56]
$\pi^+\pi^-\pi^0\pi^0\pi^0$	M3N [50]
$\omega(\to\pi^0\gamma)\pi^+\pi^-$	DM2 [38], CMD-2 [69], DM1 [70]
$\pi^+\pi^-\pi^+\pi^-\pi^+\pi^-$	M3N [50], CMD [62], DM1 [71], DM2 [72]
$\pi^+\pi^-\pi^+\pi^-\pi^0\pi^0$	M3N [50], CMD [62], DM2 [72], $\gamma\gamma2$ [56], MEA [57]
$\pi^+\pi^-\pi^0\pi^0\pi^0\pi^0$	isospin-related
$\eta\pi^+\pi^-$	DM2 [73], CMD-2 [69]
$K^+K^-\pi^0$	DM2 [74, 75]
$K^0_S\pi K$	DM1 [76], DM2 [74, 75]
K^0_SX	DM1 [77]
$\pi^+\pi^-K^+K^-$	DM2 [74]
$p\bar{p}$	FENICE [78, 79], DM2 [80, 81], DM1 [82]
$n\bar{n}$	FENICE [78, 83]
incl. (< 2 GeV)	$\gamma\gamma2$ [84], MEA [85], M3N [86], BARYON-ANTIBARYON [87]
incl. (> 2 GeV)	BES [88, 89], Crystal Ball [90, 91, 92], LENA [93], MD-1 [94], DASP [95], CLEO [96], CUSB [97], DHHM [98]

Table 1: Experiments and references for the e^+e^- data sets for the different exclusive and the inclusive channels as used in this analysis. The recent re-analysis from CMD-2 [10] supersedes

channel	inclusive (1.43,2 GeV)		exclusive (1.43,2 GeV)	
	$a_\mu^{\rm had,LO}$	$\Delta\alpha_{\rm had}(M_Z^2)$	$a_\mu^{\rm had,LO}$	$\Delta\alpha_{\rm had}(M_Z^2)$
$\pi^0\gamma$ (ChPT)	0.13 ± 0.01	0.00 ± 0.00	0.13 ± 0.01	0.00 ± 0.00
$\pi^0\gamma$ (data)	4.50 ± 0.15	0.36 ± 0.01	4.50 ± 0.15	0.36 ± 0.01
$\pi^+\pi^-$ (ChPT)	2.36 ± 0.05	0.04 ± 0.00	2.36 ± 0.05	0.04 ± 0.00
$\pi^+\pi^-$ (data)	502.78 ± 5.02	34.39 ± 0.29	503.38 ± 5.02	34.59 ± 0.29
$\pi^+\pi^-\pi^0$ (ChPT)	0.01 ± 0.00	0.00 ± 0.00	0.01 ± 0.00	0.00 ± 0.00
$\pi^+\pi^-\pi^0$ (data)	46.43 ± 0.90	4.33 ± 0.08	47.04 ± 0.90	4.52 ± 0.08
$\eta\gamma$ (ChPT)	0.00 ± 0.00	0.00 ± 0.00	0.00 ± 0.00	0.00 ± 0.00
$\eta\gamma$ (data)	0.73 ± 0.03	0.09 ± 0.00	0.73 ± 0.03	0.09 ± 0.00
K^+K^-	21.62 ± 0.76	3.01 ± 0.11	22.35 ± 0.77	3.23 ± 0.11
$K^0_SK^0_L$	13.16 ± 0.31	1.76 ± 0.04	13.30 ± 0.32	1.80 ± 0.04
$2\pi^+2\pi^-$	6.16 ± 0.32	1.27 ± 0.07	14.77 ± 0.76	4.04 ± 0.21
$\pi^+\pi^-2\pi^0$	9.71 ± 0.63	1.86 ± 0.12	20.55 ± 1.22	5.51 ± 0.35
$2\pi^+2\pi^-\pi^0$	0.26 ± 0.04	0.06 ± 0.01	2.85 ± 0.25	0.99 ± 0.09
$\pi^+\pi^-3\pi^0$	0.09 ± 0.09	0.02 ± 0.02	1.19 ± 0.33	0.41 ± 0.10
$3\pi^+3\pi^-$	0.00 ± 0.00	0.00 ± 0.00	0.22 ± 0.02	0.09 ± 0.01
$2\pi^+2\pi^-2\pi^0$	0.12 ± 0.03	0.03 ± 0.01	3.32 ± 0.29	1.22 ± 0.11
$\pi^+\pi^-4\pi^0$ (isospin)	0.00 ± 0.00	0.00 ± 0.00	0.12 ± 0.12	0.05 ± 0.05
$K^+K^-\pi^0$	0.00 ± 0.00	0.00 ± 0.00	0.29 ± 0.07	0.10 ± 0.03
$K^0_SK^0_L\pi^0$ (isospin)	0.00 ± 0.00	0.00 ± 0.00	0.29 ± 0.07	0.10 ± 0.03
$K^0_S\pi^\mp K^\pm$	0.05 ± 0.02	0.01 ± 0.00	1.00 ± 0.11	0.33 ± 0.04
$K^0_L\pi^\mp K^\pm$ (isospin)	0.05 ± 0.02	0.01 ± 0.00	1.00 ± 0.11	0.33 ± 0.04
$K\bar{K}\pi\pi$ (isospin)	0.00 ± 0.00	0.00 ± 0.00	3.63 ± 1.34	1.33 ± 0.48
$\omega(\to\pi^0\gamma)\pi^0$	0.64 ± 0.02	0.12 ± 0.00	0.83 ± 0.03	0.17 ± 0.01
$\omega(\to\pi^0\gamma)\pi^+\pi^-$	0.01 ± 0.00	0.00 ± 0.00	0.07 ± 0.01	0.02 ± 0.00
$\eta(\to\pi^0\gamma)\pi^+\pi^-$	0.07 ± 0.01	0.02 ± 0.00	0.49 ± 0.07	0.15 ± 0.02
$\phi(\to$ unaccounted)	0.06 ± 0.06	0.01 ± 0.01	0.06 ± 0.06	0.01 ± 0.01
$p\bar{p}$	0.00 ± 0.00	0.00 ± 0.00	0.04 ± 0.01	0.02 ± 0.00
$n\bar{n}$	0.00 ± 0.00	0.00 ± 0.00	0.07 ± 0.02	0.03 ± 0.01
$J/\psi, \psi'$	7.30 ± 0.43	8.90 ± 0.51	7.30 ± 0.43	8.90 ± 0.51
$\Upsilon(1S-6S)$	0.10 ± 0.00	1.16 ± 0.04	0.10 ± 0.00	1.16 ± 0.04
inclusive R	73.96 ± 2.68	92.75 ± 1.74	42.05 ± 1.14	81.97 ± 1.53
pQCD	2.11 ± 0.00	125.32 ± 0.15	2.11 ± 0.00	125.32 ± 0.15
sum	692.38 ± 5.88	275.52 ± 1.85	696.15 ± 5.68	276.90 ± 1.77

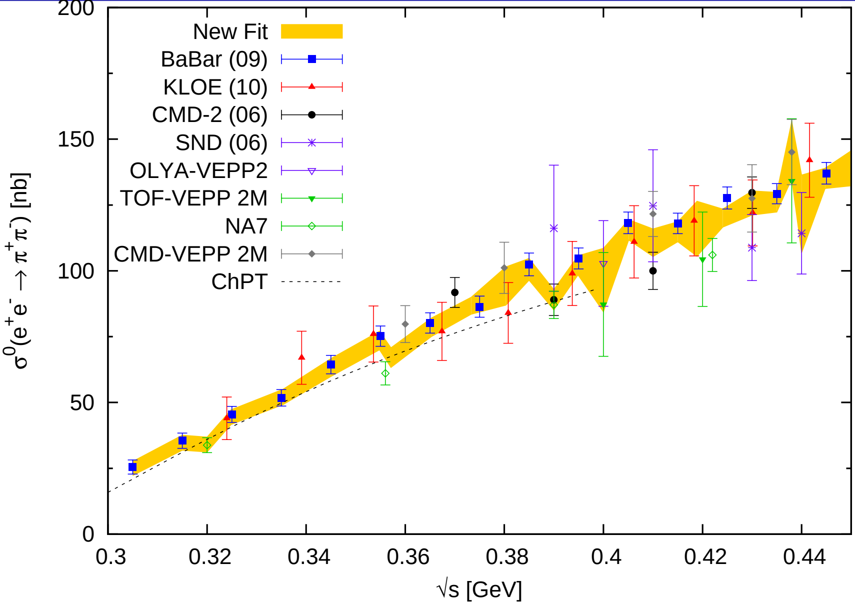
Important Channels

Contributions from various channels to a_μ(LO,had) for $\sqrt{s} < 1.8$GeV:

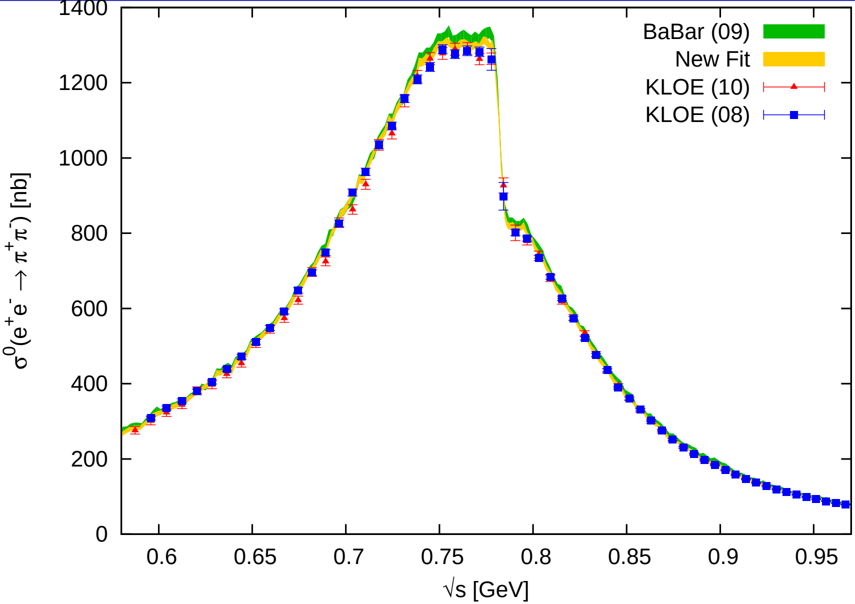
channel	HLMNT11	Davier et al '10	diff
$\pi^+\pi^-$	505.65 ± 3.09	507.80 ± 2.84	-2.15
$\pi^+\pi^-\pi^0$	47.38 ± 0.99	46.00 ± 1.48	1.38
K^+K^-	22.09 ± 0.46	21.63 ± 0.73	0.46
$\pi^+\pi^-2\pi^0$	18.62 ± 1.15	18.01 ± 1.24	0.61
$2\pi^+2\pi^-$	13.50 ± 0.44	13.35 ± 0.53	0.15
$K_S^0K_L^0$	13.32 ± 0.16	12.96 ± 0.39	0.36
$\pi^0\gamma$	4.54 ± 0.14	4.42 ± 0.19	0.12
⋮	⋮	⋮	⋮
Sum	634.28 ± 3.53	633.93 ± 3.61	0.35

table taken from HLMNT11

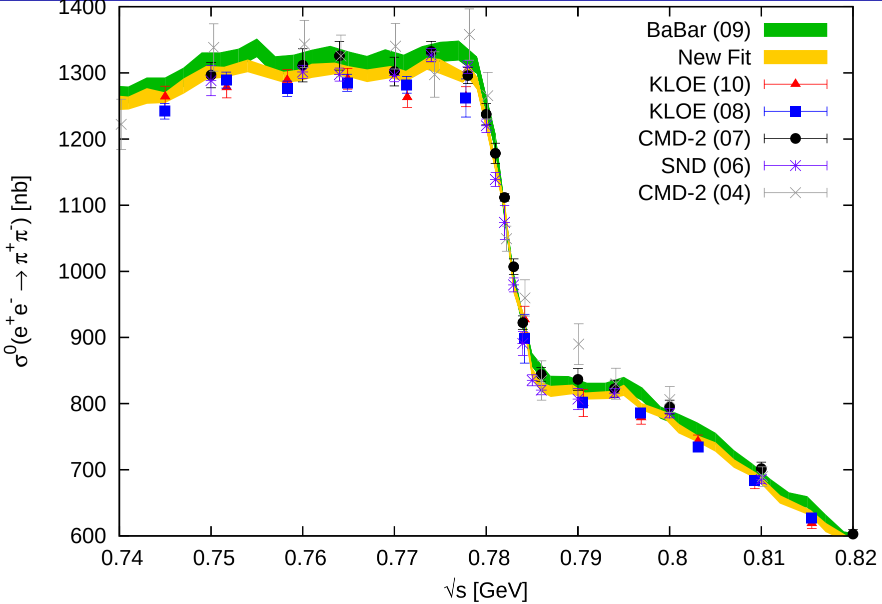
$\pi^+\pi^-$ channel: Low Energy Tail



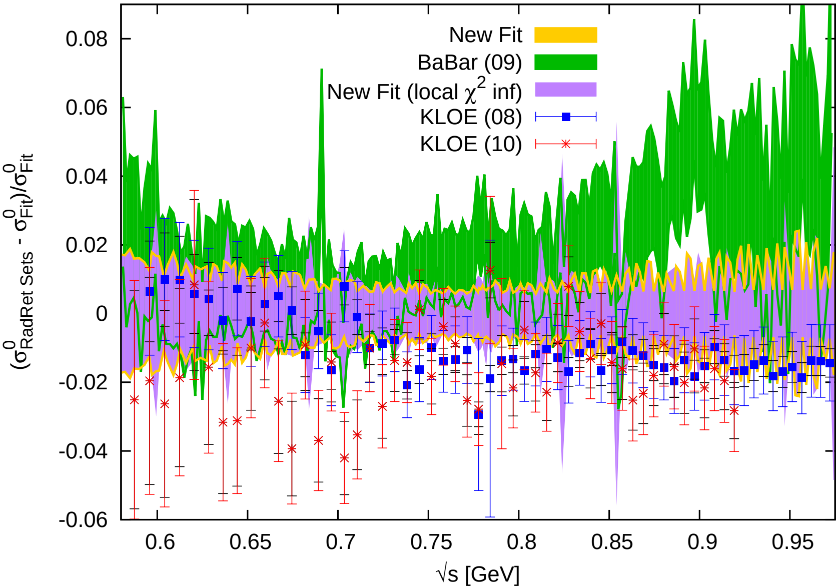
$\pi^+\pi^-$ channel: New Radiative Return Data



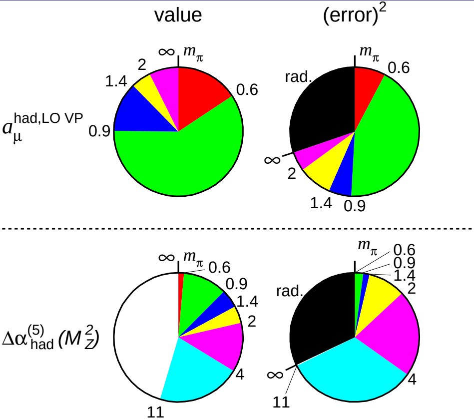
$\pi^+\pi^-$ channel: Zoom-In at ρ-ω Region



Rad. Rtn. Data (for $\pi^+\pi^-$) and Our Combined Result



Results: $a_\mu^{\rm had,LO}$, $a_\mu^{\rm had,NLO}$ and $\Delta\alpha_{\rm had}^{(5)}(M_Z^2)$

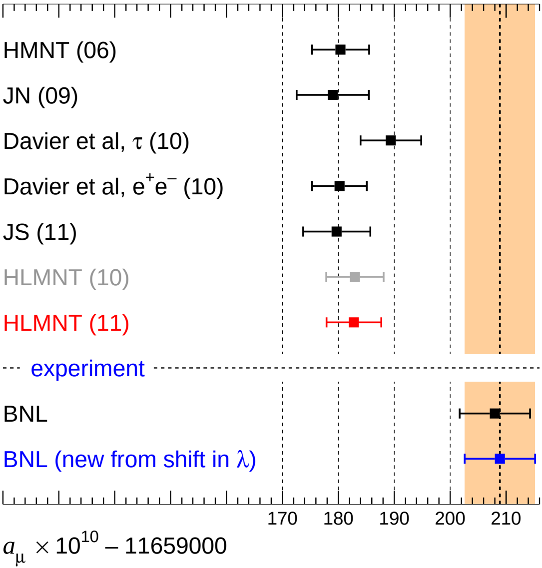


$$a_\mu^{\rm had,LO} =(694.91 \pm 3.72_{\rm exp} \pm 2.10_{\rm rad}) \times 10^{-10}$$

$$a_\mu^{\rm had,NLO} =(-9.84 \pm 0.06_{\rm exp} \pm 0.04_{\rm rad}) \times 10^{-10}$$

$$\Delta\alpha_{\rm had}^{(5)}(M_Z^2) =(276.26 \pm 1.16_{\rm exp} \pm 0.74_{\rm rad}) \times 10^{-4}$$

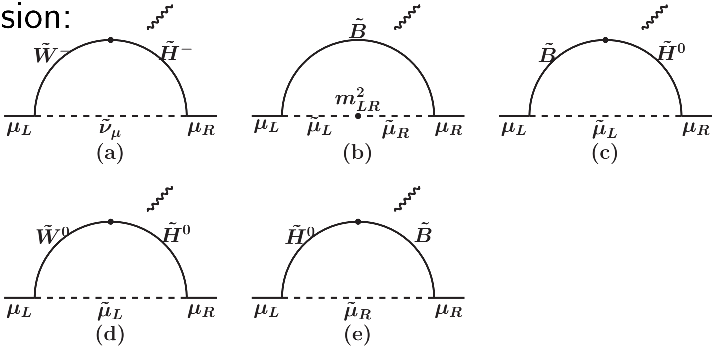
Full SM Result and Comparison with Other Groups



SUSY Contributions to Muon $g-2$

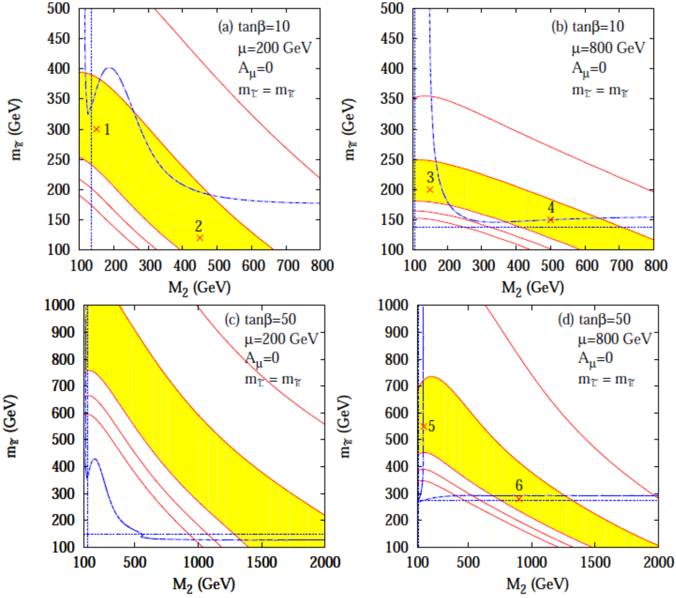
Suppose that the 3.3 σ deviation is due to SUSY...

Leading **SUSY contributions** in the $m_Z/m_{\rm SUSY}$ expansion:



In most cases, the $\tilde{\chi}^{\pm}$-$\tilde{\nu}$ diagram (a) and/or the $\widetilde{B}$-$\widetilde{\mu}_{L/R}$ diagram (b) dominate. (Lopez-Nanopoulos-Wang, Chattopadhyay-Nath, Moroi, $\cdots$)

MSSM Contributions to Muon $g-2$



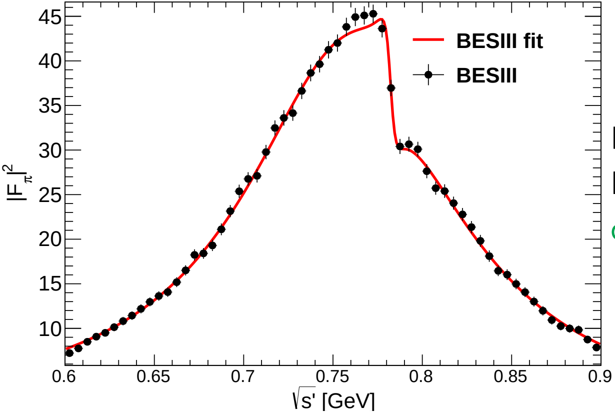
x-axis: M_2
(gaugino mass)

y-axis: $m_{\tilde{l}}$
(slepton mass)

Figs from Cho, Hagiwara, Matsumoto and DN

Very recent updates

BES-III new pion form factor data (1)



Data taken by using the ISR technique $\longrightarrow$ For details, see next talk

Fig. from M. Ablikim et al (BESIII Collaboration), arXiv:1507.08188v2

BES-III new pion form factor data (2)

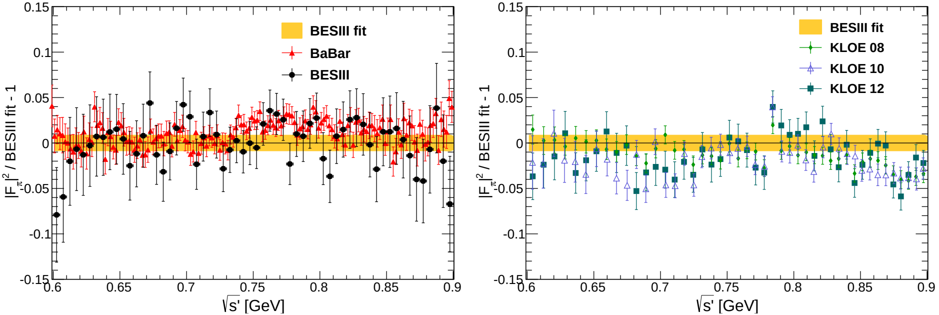
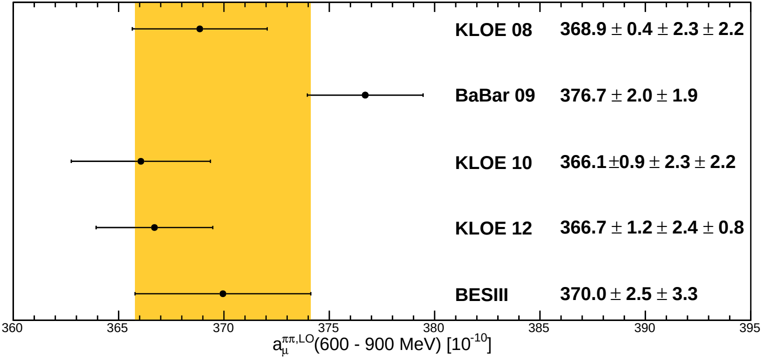


Fig. from M. Ablikim et al (BESIII Collaboration), arXiv:1507.08188v2

BES-III new pion form factor data (3)



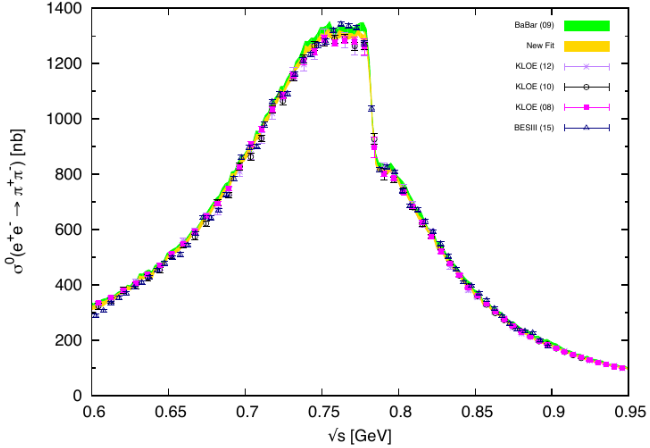
Prediction from BES-III data 'interpolate' those from KLOE and BaBar, but closer to KLOE

Fig. from M. Ablikim et al (BESIII Collaboration), arXiv:1507.08188v2

$\pi^+\pi^-$: Effect of KLOE12 + BESIII data in `global' 2π fit

Very preliminary first results (this Monday) from my PhD student Alex Kheshavarzi:

ISR data comparison in ρ peak region and new fit band:



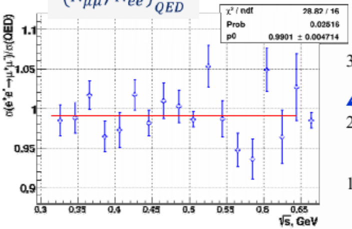
Slide by T. Teubner (Liverpool), talk at 'High-precision QCD at low energy,' Aug. '15

Near future (5-7yrs) prospects

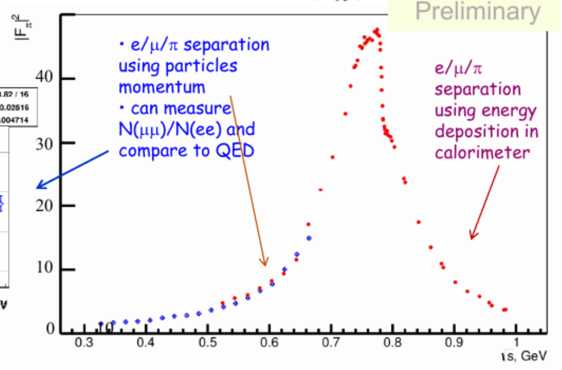
$e^+e^- \to \pi^+\pi^-$

- Gives largest contribution to $(g\text{-}2)_\mu$ calculation (65% of δa_μ)
- Goal is to reach systematic uncertainty at the level of 0.35%

$$\frac{(N_{\mu\mu}/N_{ee})_{exp}}{(N_{\mu\mu}/N_{ee})_{QED}}$$



$|F_\pi|^2$



Slide by V. B. Golubev (BINP), talk at "Tau 2014", Sept. 2014

Near Future Prospects (Blum et al, arXiv:1311.2198)

Current status and near-future ($\sim$ 5-7 yrs) improvements in δa_μ (in units of 10^{-11})

Error	[20]	[21]	Future
$\delta a_\mu^{\rm SM}$	49	50	35
$\delta a_\mu^{\rm HLO}$	42	43	26
$\delta a_\mu^{\rm HLbL}$	26	26	25
$\delta(a_\mu^{\rm EXP} - a_\mu^{\rm SM})$	80	80	40

[20]: DHMZ
[21]: HLMNT

Near-future improvements in $\delta a_\mu^{\rm HLO}$ mainly from VEPP-2000 and BES-III.

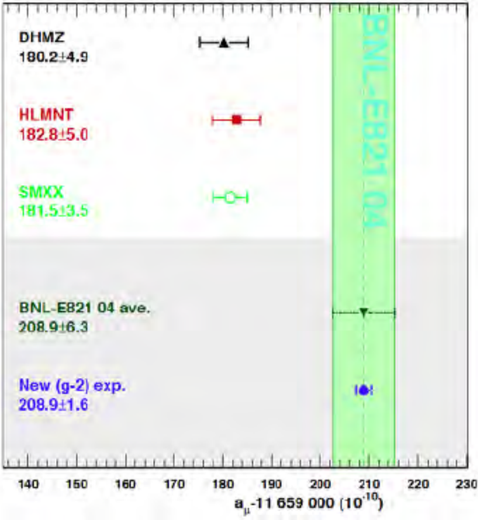


Table and Fig. from T. Blum et al, arXiv:1311.2198

Near Future Prospects, Lattice (Blum et al)

As for the hadronic vacuum polarization (HVP),

"...the lattice-QCD uncertainty on a_μ(HVP), currently at the 5%-level, can be reduced to 1 or 2% within the next few years."

"With increasing experience and computer power, it should be possible to compete with the e^+e^- determination of a_μ(HVP) by the end of the decade, perhaps sooner with additional technical advances."

As for l-by-l,

"... a lattice calculation with even a solid 30% error would already be very interesting. Such a result, ..., is not out of the question during the next 3-5 years."

From T. Blum et al, arXiv:1311.2198

Byproducts

★ $\alpha(M_Z^2)$: the least well known among $\{G_\mu, M_Z, \alpha(M_Z^2)\}$, which are used as **input** to precision electroweak fits.

★ Running of α

$$\alpha(M_Z^2) = \frac{\alpha}{1 - \Delta\alpha_{\rm lep}(M_Z^2) - \Delta\alpha_{\rm had}^{(5)}(M_Z^2) - \Delta\alpha^{\rm top}(M_Z^2)}$$

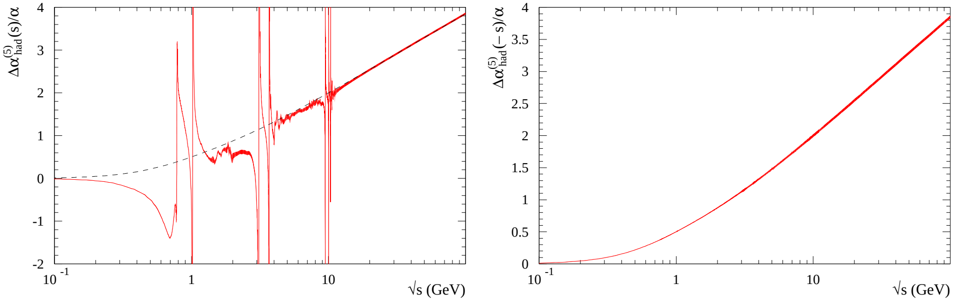
where $\Delta\alpha_{\rm lep}(M_Z^2) = 0.03149769$ (Steinhauser), $\Delta\alpha^{\rm top}(M_Z^2) = -\ 0.0000728(14)$ and $\alpha = 1/137.035999679(94)$ (PDG10).

★ Similar dispersion relation: ($\Longrightarrow$ **byproduct** of $a_\mu^{\rm had,LO}$)

$$\Delta\alpha_{\rm had}^{(5)}(s) = -\frac{\alpha s}{3\pi} \mathrm{P} \int \frac{R(s')ds'}{s'(s'-s)}$$

★ Our results:
$$\Delta\alpha_{\rm had}^{(5)}(M_Z^2) = (276.3 \pm 1.4) \times 10^{-4},$$
$$\alpha(M_Z^2)^{-1} = 128.944 \pm 0.019.$$

The hadronic contribution $\Delta\alpha^{(5)}_{\rm had}(q^2)$ to the running QED coupling for $q^2 > 0$ (left) and $q^2 < 0$ (right)



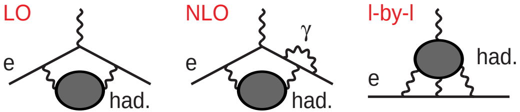
Fortran subroutine to compute the above is available from us upon request

Byproduct: Standard Model prediction for electron $g-2$

QED contribution	$115\ 965\ 218\ 00.7\ (0.6)_{4loop}(0.4)_{5loop}(7.6)_{\alpha}$ (using $\alpha(^{87}\mathrm{Rb})$)	Aoyama et al
EW contribution	0.2973 (0.0052)	Czarnecki et al
Hadronic contributions		
LO hadronic	**18.66 (0.11)**	**DN & TT** (was $18.75(0.18)$ Davier et al '98)
NLO hadronic	**$-$2.23 (0.01)**	**DN & TT** (was $-2.25(0.05)$ Krause '97)
light-by-light	0.39 (0.13)	Jegerlehner+Nyffeler
Theory total	**$115\ 965\ 218\ 17.8\ (0.6)_{4loop}(0.4)_{5loop}(0.2)_{had}(7.6)_{\alpha}$**	
Experiment	**115 965 218 07.3 (2.8)**	Gabrielse et al
Theory − Exp	**10.5 (8.1)**	**1.3σ discrepancy**

(in units of 10^{-13}. Numbers taken from Giudice et al, JHEP**11**(2012)113)

n.b.: hadronic contributions:



Summary

- Hadronic contrib. to the muon $g-2$: key to improve the Standard Model prediction
- $\gtrsim 3\,\sigma$ discrepancy in $(g-2)_\mu$ between experiment and theory
  $\Longrightarrow$ **New physics?** "Light" new particles like $\widetilde{\mu}, \mu_{\mathrm{KK}}, \ldots$???
  $\Longrightarrow$ Worth studying μ-EDM, $\mu \to e\gamma$, μ-e conv., . . . !
  ($\Leftrightarrow$ No new physics seen at the LHC so far. What does this mean?)
- Two new experiments to measure the muon $g-2$ planned at J-PARC and Fermilab.
- To establish this discrepancy more firmly, important to resolve the tension between the KLOE and BaBar data
  $\Longrightarrow$ new data from BES-III just appeared!
  $\Longrightarrow$ new precise data from VEPP-2000 and SuperKEKB strongly awaited.