Parton distributions at the LHC

Sven-Olaf Moch

Universität Hamburg & DESY, Zeuthen

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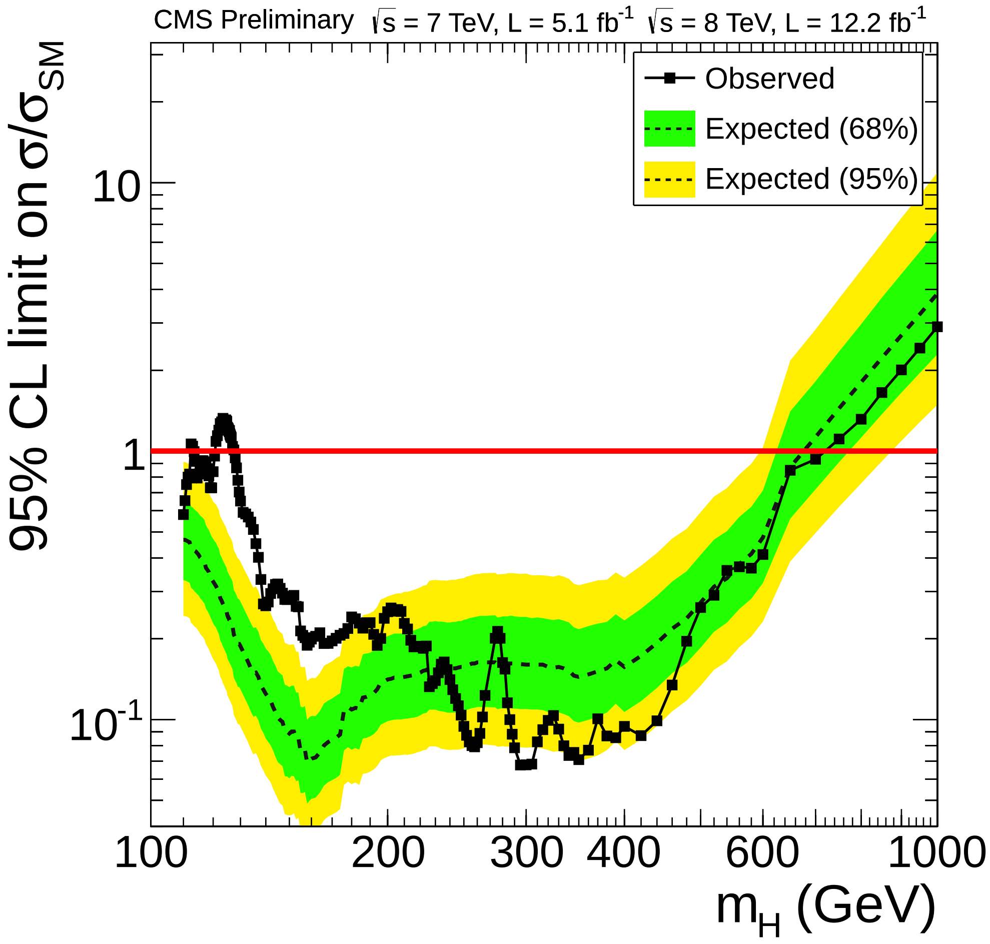
Plan

- Cross sections in perturbative QCD
- Non-perturbative input parameters
  - parton distributions
  - strong coupling $\alpha_s(M_Z)$
  - heavy quark masses
- Constraints from LHC measurements
  - $W^{\pm}$- and Z-boson production

Example from LHC Higgs measurements

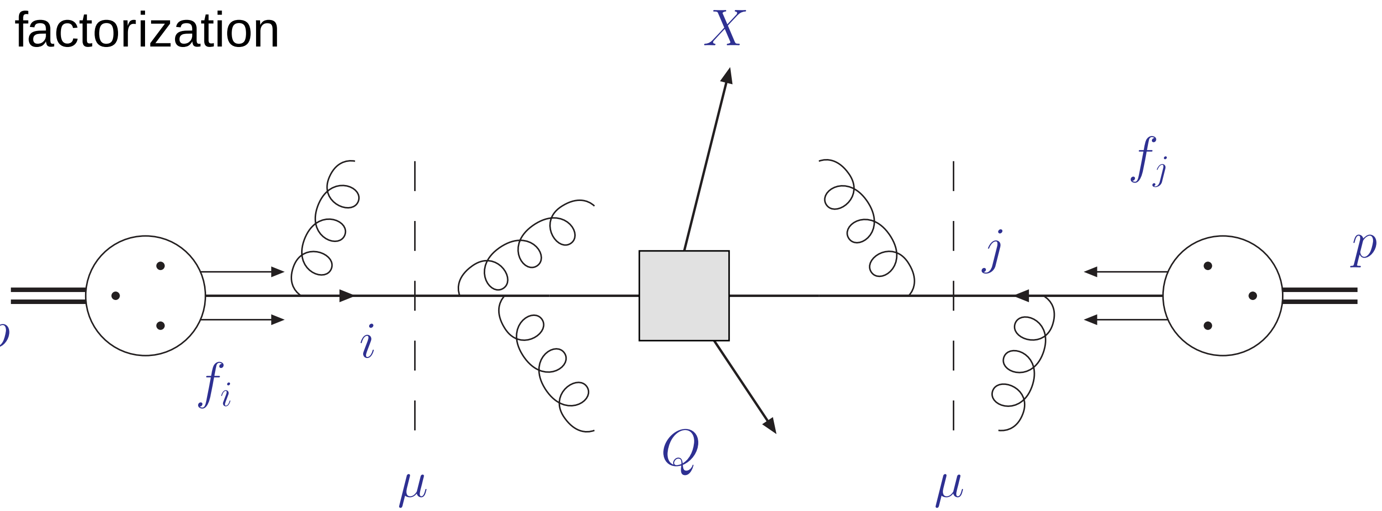
CMS coll. Dec 2012



- Signal strength of all analyzed decay modes
  - normalization to Standard Model expectation
  - accuracy of $\sigma_{\rm SM}$ crucial

QCD factorization

- QCD factorization

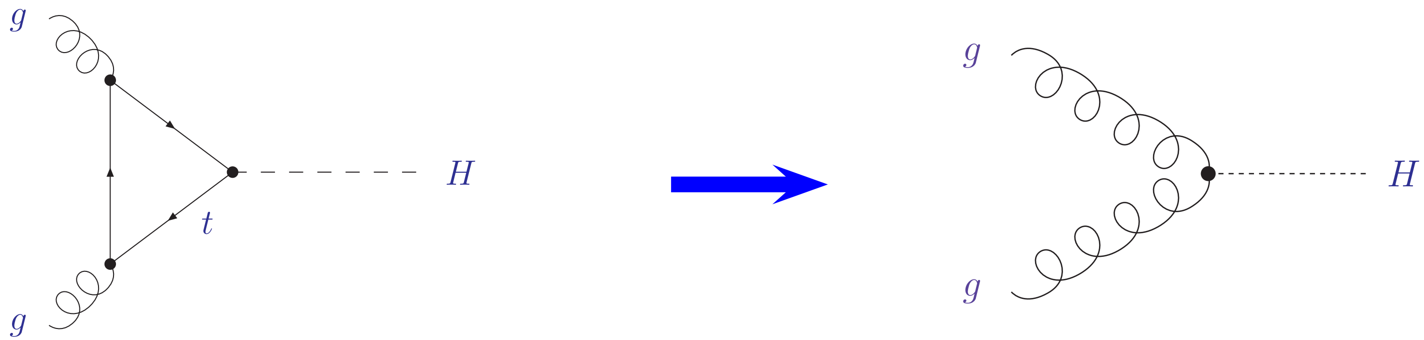


$$\sigma_{pp\to X} = \sum_{ij} f_i(\mu^2) \otimes f_j(\mu^2) \otimes \hat{\sigma}_{ij\to X}\left(\alpha_s(\mu^2), Q^2, \mu^2, m_X^2\right)$$

- Hard parton cross section $\hat{\sigma}_{ij\to X}$ calculable in perturbation theory
  - known to NLO, NNLO, ... ($\mathcal{O}(\text{few}\%)$ theory uncertainty)
- Non-perturbative parameters: parton distribution functions f_i, strong coupling α_s, particle masses m_X
  - known from global fits to exp. data, lattice computations, ...

Higgs production in gg-fusion

Effective theory



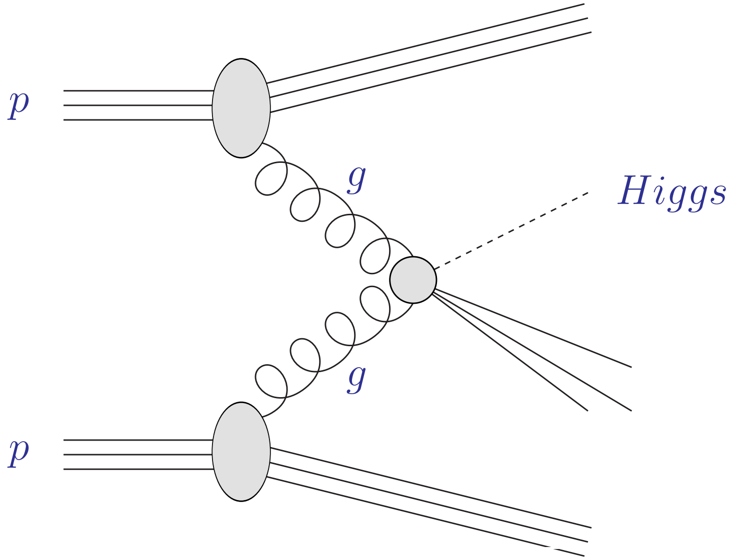
- Integration of top-quark loop (finite result)
  - decay width $H \to gg$ ($m_q = 0$ for light quarks, m_t heavy)

$$\Gamma_{H \to gg} \;=\; \frac{G_\mu\, m_H^3}{64\,\sqrt{2}\,\pi^3}\, \alpha_s^2\; f\!\left(\frac{m_H^2}{4m_t^2}\right)$$

- Effective theory in limit $m_t \to \infty$; Lagrangian $\mathcal{L} = -\frac{1}{4}\,\frac{H}{v}\,C_H\,G^{\mu\nu\,a}\,G^a_{\mu\nu}$
  - operator $HG^{\mu\nu\,a}\,G^a_{\mu\nu}$ relates to stress-energy tensor
  - additional renormalization proportional to QCD β-function required

    Kluberg-Stern, Zuber '75; Collins, Duncan, Joglekar '77

QCD corrections to ggF



- Hadronic cross section $\sigma_{pp\to H}$ with $\tau = m_H^2/S$
  - renormalization/factorization (hard) scale $\mu = \mathcal{O}(m_H)$

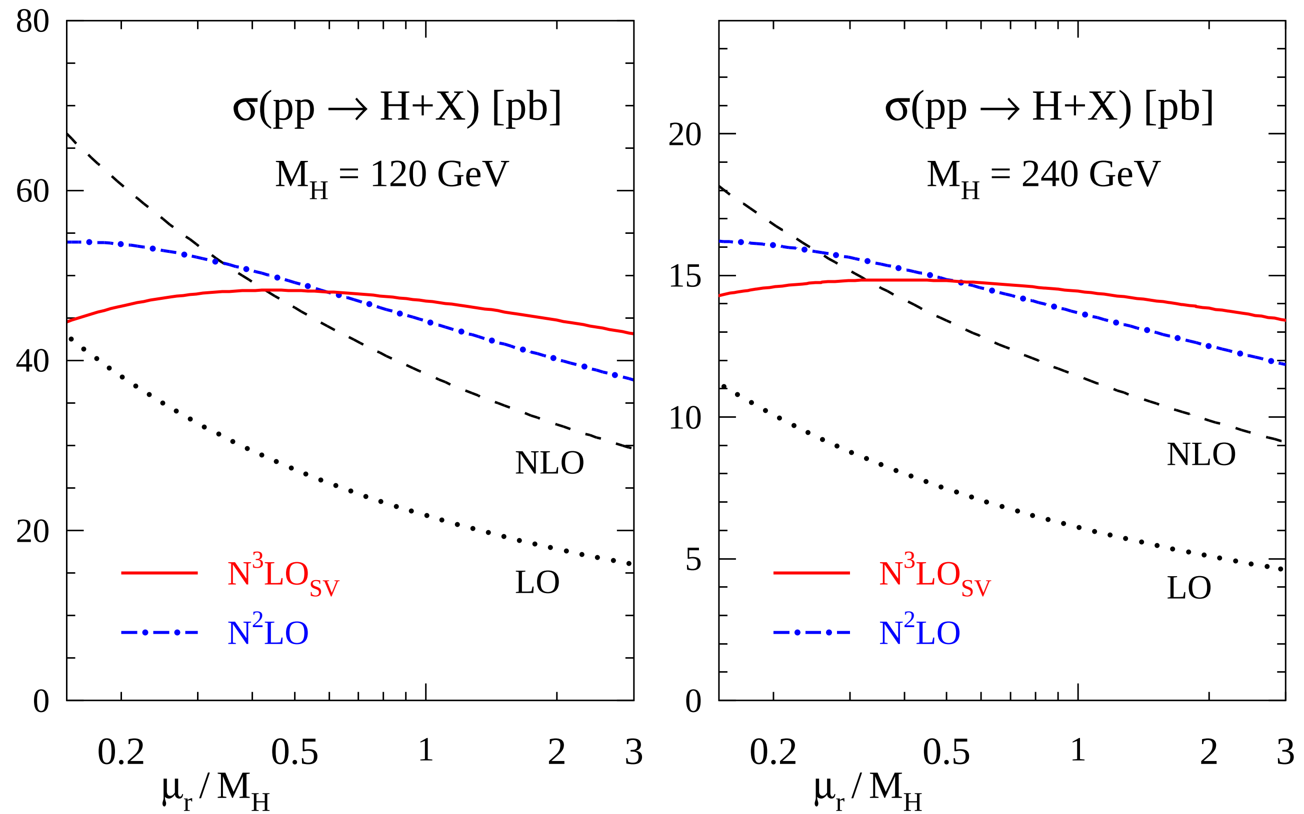
$$\sigma_{pp\to H} = \sum_{ij} \int_\tau^1 \frac{dx_1}{x_1} \int_{x_1}^1 \frac{dx_2}{x_2} f_i\left(\frac{x_1}{x_2}, \mu^2\right) f_j\left(x_2, \mu^2\right) \hat{\sigma}_{ij\to H}\left(\frac{\tau}{x_1}, \frac{\mu^2}{m_H^2}, \alpha_s(\mu^2)\right)$$

- Partonic cross section $\hat{\sigma}_{ij\to H}$

$$\hat{\sigma}_{ij\to H} = \underbrace{\alpha_s^2\left[\hat{\sigma}^{(0)}_{ij\to H} + \alpha_s\,\hat{\sigma}^{(1)}_{ij\to H}\right.}_{} + \left.\alpha_s^2\,\hat{\sigma}^{(2)}_{ij\to H} + \dots\right]$$

NLO: standard approximation (large uncertainties)

Perturbation theory at work



- Apparent convergence of perturbative expansion
  - NNLO corrections still large
    Harlander, Kilgore '02; Anastasiou, Melnikov '02; Ravindran, Smith, van Neerven '03
  - improvement through complete soft N^3LO corrections S.M., Vogt '05
    or NNLL resummtion Catani, de Florian, Grazzini, Nason '03, Ahrens et al. '10
- Perturbative stability under renormalization scale variation

Non-perturbative parameters

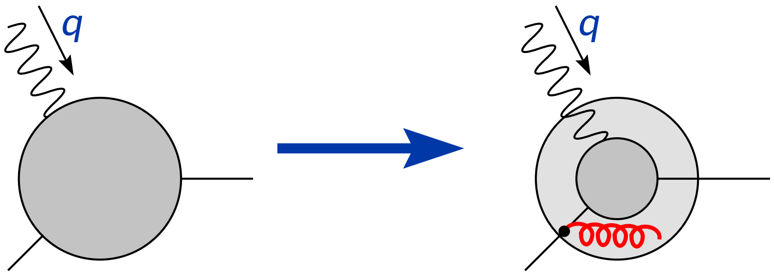
Input for collider phenomenology

- Non-perturbative parameters are universal
- Determination from comparision to experimental data
  - masses of heavy quarks m_c, m_b, m_t
  - parton distribution functions $f_i(x, \mu^2)$
  - strong coupling constant $\alpha_s(M_Z)$

Interplay with perturbation theory

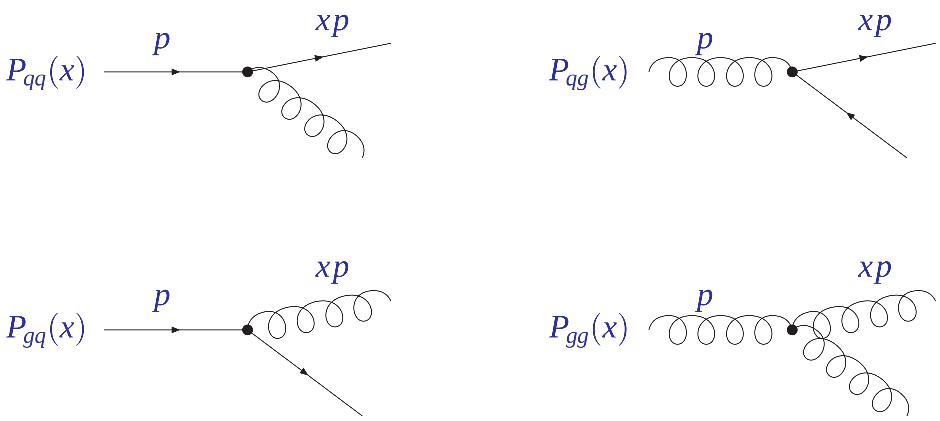
- Accuracy of determination driven by precision of theory predictions
- Non-perturbative parameters sensitive to
  - radiative corrections at higher orders
  - renormalization and factorization scales μ_R, μ_F
  - chosen scheme (e.g. ($\overline{MS}$ scheme)
  - ...

Parton evolution

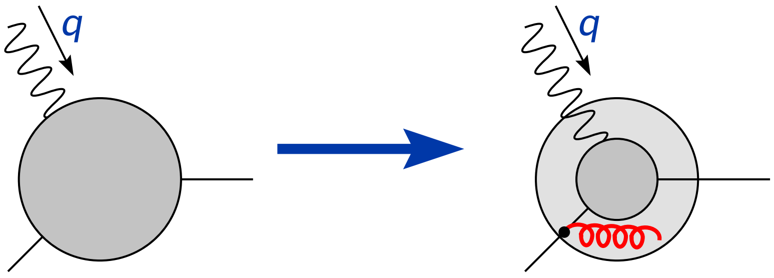


- Proton in resolution $1/Q \longrightarrow$ sensitive to lower momentum partons

- Feynman diagrams in leading order



Parton evolution



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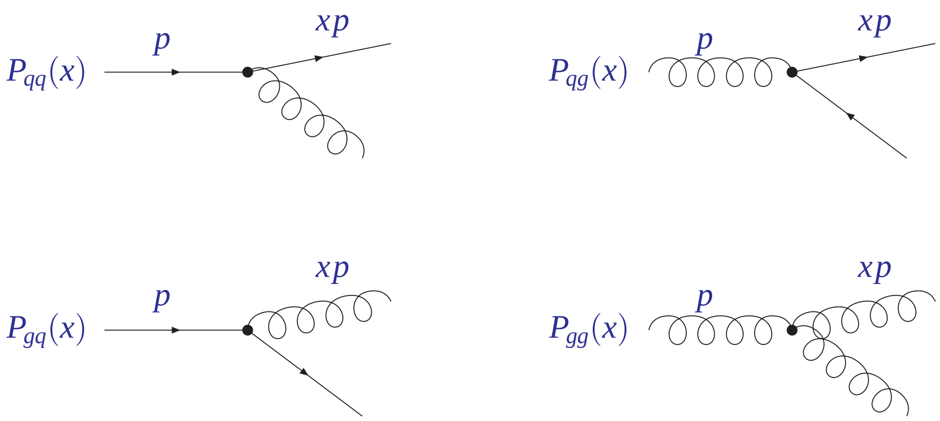

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- Evolution equations for parton distributions f_i
  - predictions from fits to reference processes (universality)

$$\frac{d}{d \ln \mu^2} f_i(x, \mu^2) = \sum_k \left[P_{ik}(\alpha_s(\mu^2)) \otimes f_k(\mu^2) \right] (x)$$

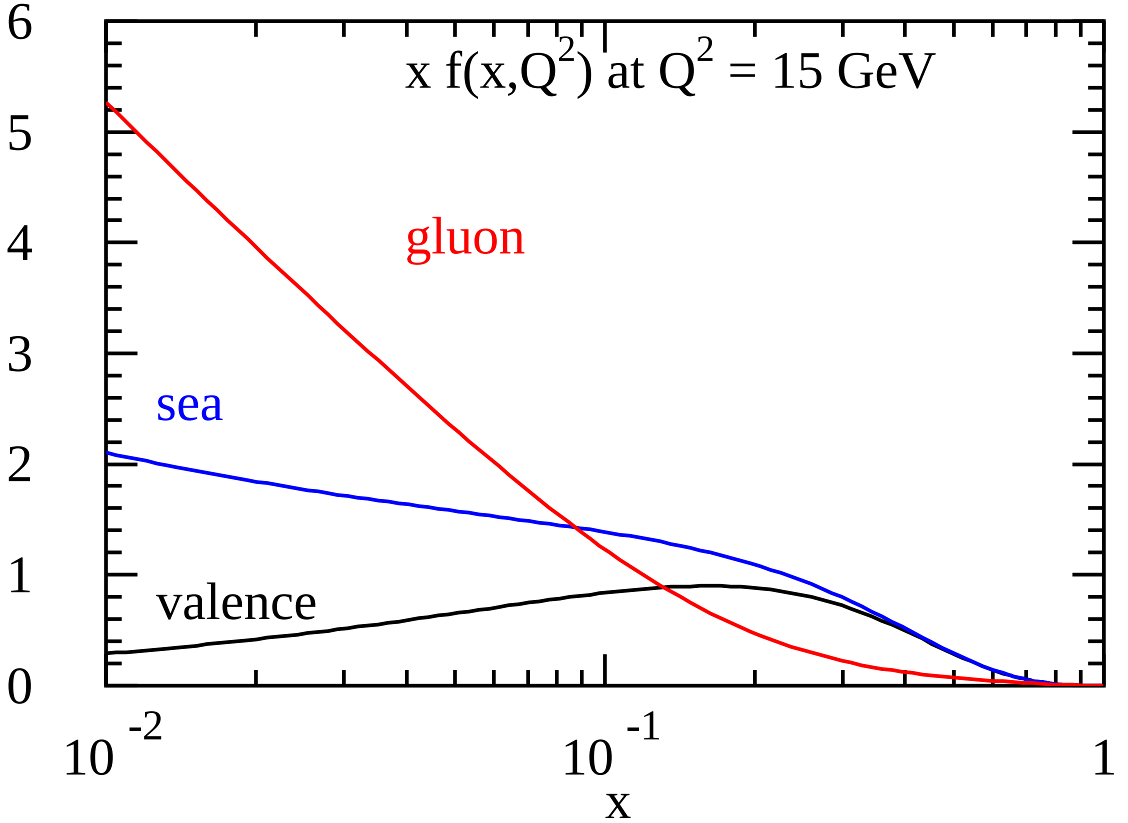
- Splitting functions P

$$P = \underbrace{\alpha_s P^{(0)} + \alpha_s^2 P^{(1)}}_{} + \alpha_s^3 P^{(2)} + \ldots$$

NLO: standard approximation (large uncertainties)

Parton distributions in proton

- Valence $q - \bar{q}$ (additive quantum numbers) sea (part with $q + \bar{q}$)



- Parameterization (bulk of data from deep-inelastic scattering)
  - structure function $F_2 \longrightarrow$ quark distribution
  - scale evolution (perturbative QCD) $\longrightarrow$ gluon distribution

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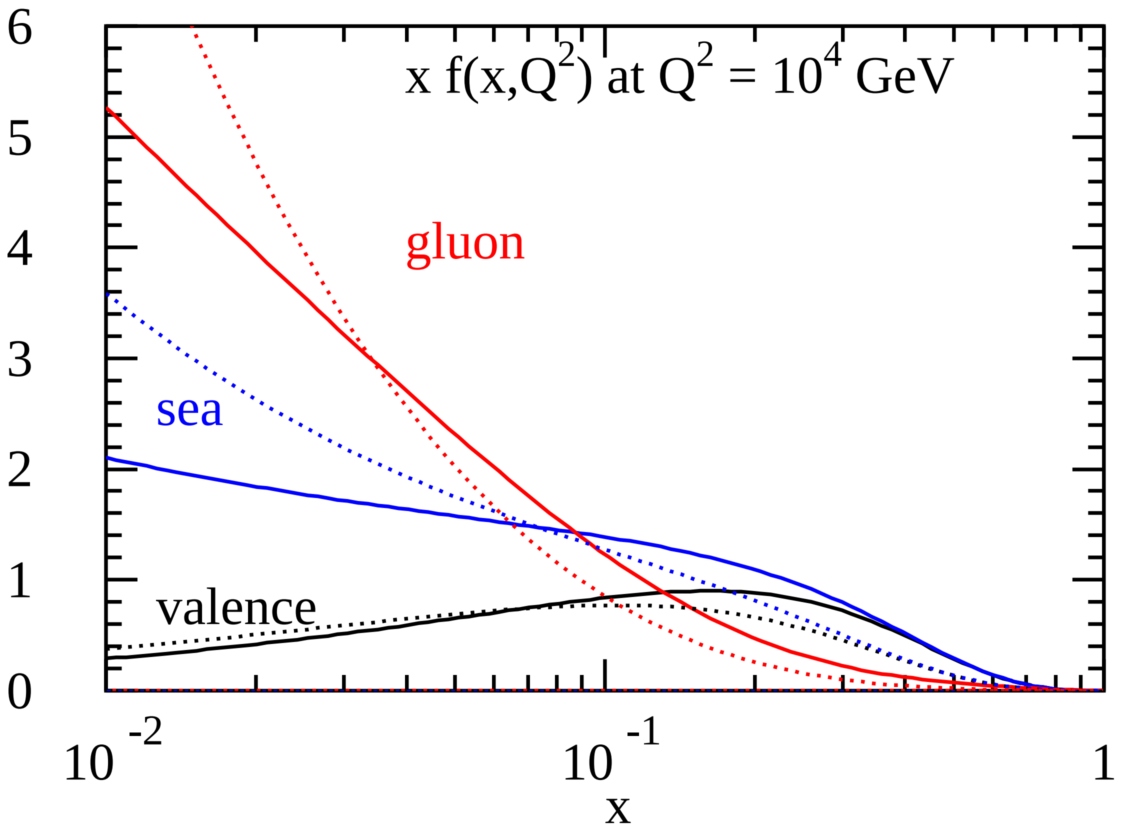

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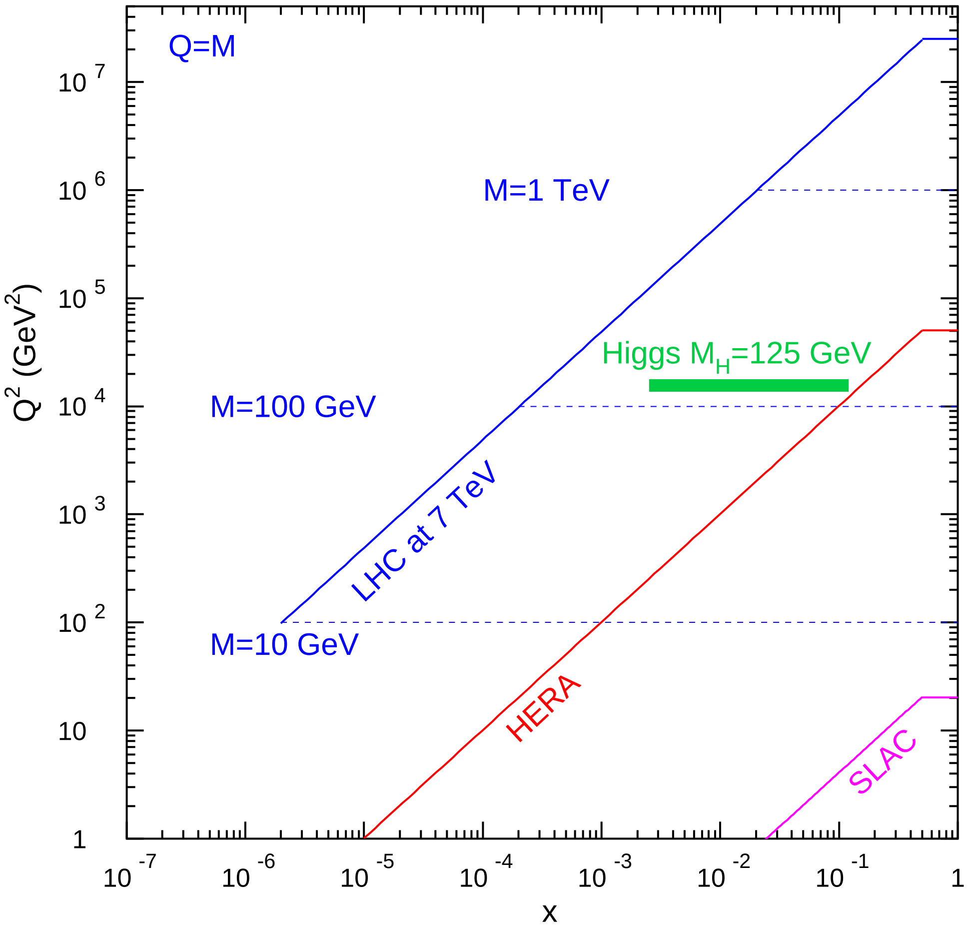
Parton luminosity at LHC



- LHC run at $\sqrt{s}$ = 7/8 TeV
  - parton kinematics well covered by HERA and fixed target experiments
- Parton kinematics at effective $\langle x \rangle = M/\sqrt{S}$
  - 100 GeV physics: small-x, sea partons
  - TeV scales: large-x

Parton distribution fits

Example

- ABM PDF set Alekhin, Blümlein, S.M. ‘12

Theory considerations

- Consistent theory description for consistent data sets
- Determination of PDFs and strong coupling constant α_s to NNLO QCD
- Consistent scheme for treatment of heavy quarks
  - fixed-flavor number scheme for $n_f = 3, 4, 5$
  - $\overline{\mathrm{MS}}$-scheme for quark masses and α_s
- Full account of error correlations

Data considered in the fit

- Analysis of world data for deep-inelastic scattering and fixed-target data for Drell-Yan process
  - inclusive DIS data HERA, BCDMS, NMC, SLAC
  - Drell-Yan data (fixed target) E-605, E-866
  - neutrino-nucleon DIS data (di-muon production) CCFR/NuTeV

Iterative cycle of PDF fits

i) check of compatibility of new data set with available world data

ii) study of potential constraints due to addition of new data set to fit

iii) perform high precision measurement of the non-perturbative parameters

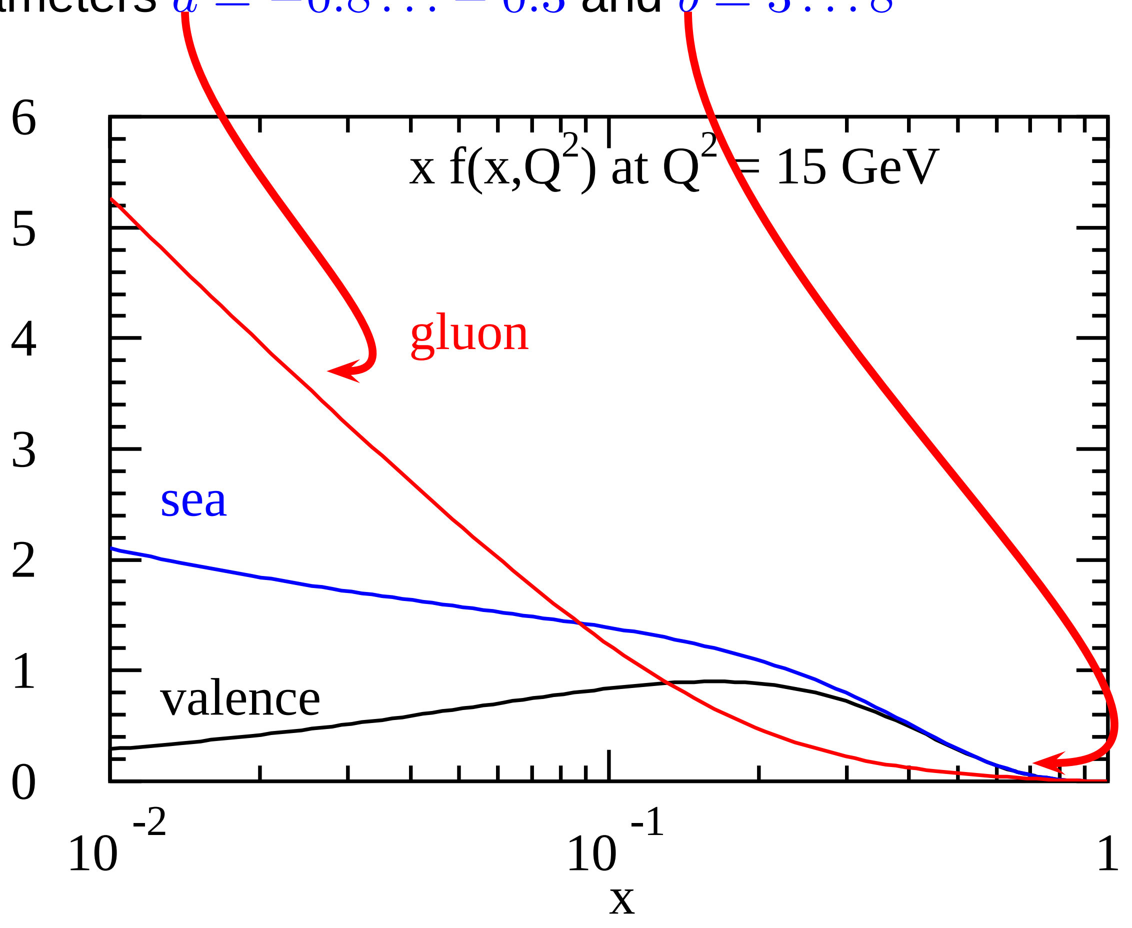
- parton distributions
- strong coupling $\alpha_s(M_Z)$
- heavy quark masses

PDF ansatz

- Parameterization at low scale Q_0 for sea-quarks

$$xf(x, Q_0^2) \quad \simeq \quad x^a \, (1-x)^b \ ,$$

  - parameters $a = -0.8 \ldots - 0.3$ and $b = 3 \ldots 8$



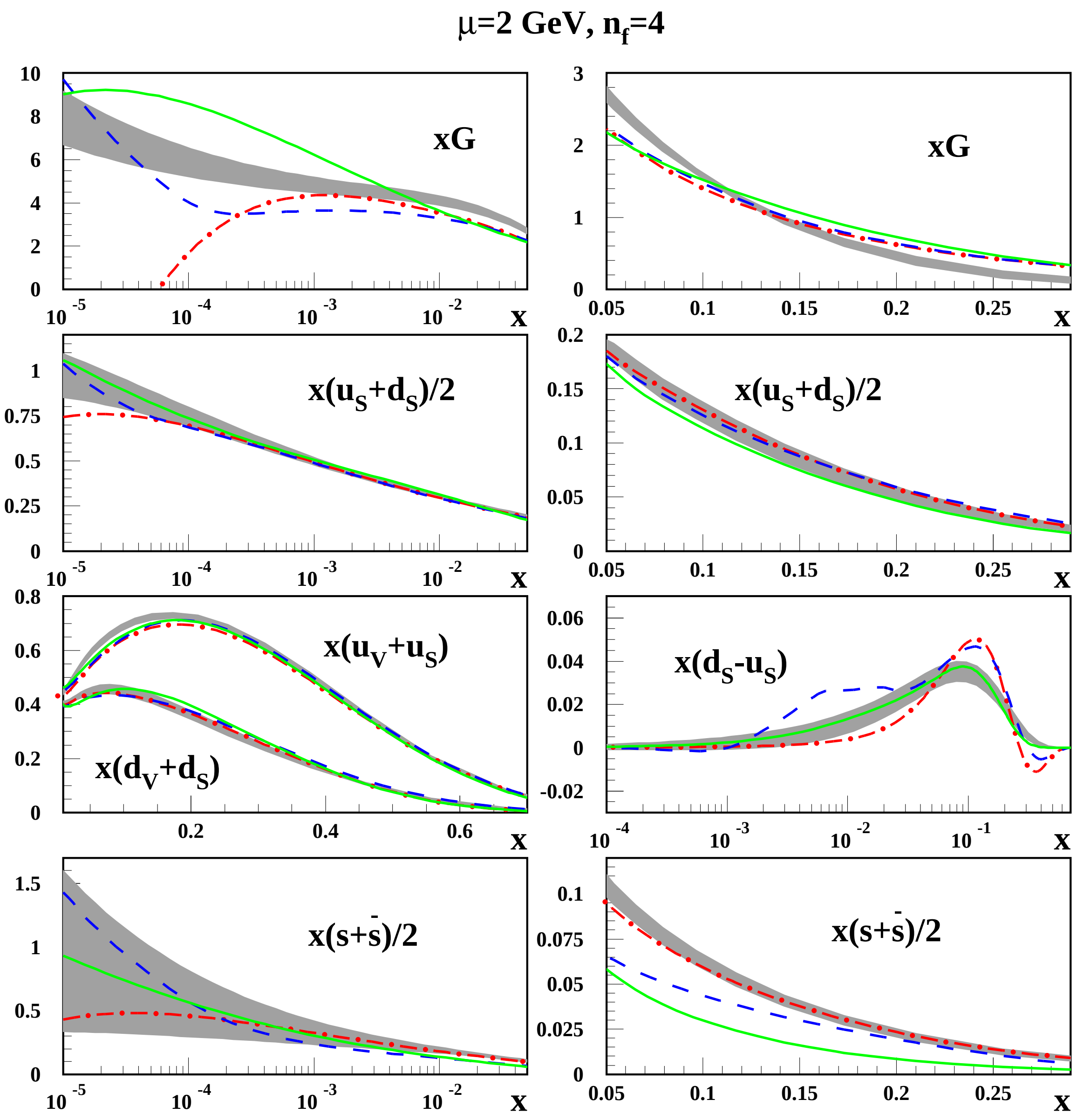
PDF ansatz (details)

- ABM PDFs parameterized at scale $Q_0 = 3\mathrm{GeV}$ in scheme with $n_f = 3$
  Alekhin, Blümlein, S.M. '12
  - ansatz for valence-/sea-quarks, gluon with polynomial $P(x)$
  - strange quark is taken in charge-symmetric form
  - 24 parameters in polynomials $P(x)$
  - 4 additional fit parameters: $\alpha_s^{(n_f=3)}(\mu = 3$ GeV), m_c, m_b and deuteron correction

$$
\begin{aligned}
xq_v(x,Q_0^2) &= \frac{2\delta_{qu}+\delta_{qd}}{N_q^v} x^{a_q}(1-x)^{b_q} x^{P_{qv}(x)} \\
xu_s(x,Q_0^2) = x\bar{u}_s(x,Q_0^2) &= A_{us} x^{a_{us}}(1-x)^{b_{us}} x^{a_{us} P_{us}(x)} \\
x\Delta(x,Q_0^2) = xd_s(x,Q_0^2) - xu_s(x,Q_0^2) &= A_\Delta x^{a_\Delta}(1-x)^{b_\Delta} x^{P_\Delta(x)} \\
xs(x,Q_0^2) = x\bar{s}(x,Q_0^2) &= A_s x^{a_s}(1-x)^{b_s}\,, \\
xg(x,Q_0^2) &= A_g x^{a_g}(1-x)^{b_g} x^{a_g P_g(x)}
\end{aligned}
$$

- Ansatz provides sufficient flexibility; no additional terms required to improve the quality of fit

Parton distributions for the LHC



- 1σ band for ABM11 PDFs (NNLO, 4-flavors) at $\mu = 2$ GeV
  Alekhin, Blümlein, S.M.'12
- comparison with:
  JR09 (solid lines),
  MSTW (dashed dots) and
  NN21 (dashes)
- Some interesting observations to be made . . .

Quality of fit

	Experiment	NDP	$\chi^2(\mathrm{NNLO})$	$\chi^2(\mathrm{NLO})$
DIS inclusive	H1&ZEUS	486	537	531
	H1	130	137	132
	BCDMS	605	705	695
	NMC	490	665	661
	SLAC-E-49a	118	63	63
	SLAC-E-49b	299	357	357
	SLAC-E-87	218	210	219
	SLAC-E-89a	148	219	215
	SLAC-E-89b	162	133	132
	SLAC-E-139	17	11	11
	SLAC-E-140	26	28	29
Drell-Yan	FNAL-E-605	119	167	167
	FNAL-E-866	39	52	55
DIS di-muon	NuTeV	89	46	49
	CCFR	89	61	62
Total		3036	3391	3378

Covariance matrix

- Correlations of PDF fit parameters (I)

	a_u	b_u	$\gamma_{1,u}$	$\gamma_{2,u}$	a_d	b_d	A_Δ	b_Δ	A_{us}	a_{us}	b_{us}	a_g	b_g	$\gamma_{1,g}$
a_u	1.0000	0.9692	0.9787	-0.7929	0.7194	0.5279	-0.1460	-0.1007	0.7481	0.6835	-0.4236	-0.2963	0.3391	0.3761
b_u		1.0000	0.9396	-0.7244	0.6792	0.4939	-0.1146	-0.1099	0.7404	0.6840	-0.4146	-0.3138	0.3464	0.3738
$\gamma_{1,u}$			1.0000	-0.8940	0.6506	0.4646	-0.1865	-0.0539	0.6728	0.6093	-0.4799	-0.2755	0.3441	0.3717
$\gamma_{2,u}$				1.0000	-0.4102	-0.2267	0.2357	-0.0182	-0.4075	-0.3495	0.4543	0.1713	-0.3156	-0.3149
a_d					1.0000	0.8827	-0.2155	-0.1964	0.6875	0.6435	-0.3030	-0.3354	0.2635	0.3500
b_d						1.0000	-0.2462	-0.0979	0.5359	0.5099	-0.2957	-0.3443	0.3157	0.3763
A_Δ							1.0000	-0.2068	-0.0689	-0.0698	0.2381	-0.0168	0.0384	0.0453
b_Δ								1.0000	0.1015	0.1279	-0.4146	-0.0852	-0.1185	-0.0892
A_{us}									1.0000	0.9884	-0.4678	-0.4679	0.1961	0.2504
a_{us}										1.0000	-0.4520	-0.5195	0.1982	0.2596
b_{us}											1.0000	0.1436	0.0444	-0.0180
a_g												1.0000	-0.6289	-0.7662
b_g													1.0000	0.9392
$\gamma_{1,g}$														1.0000

Covariance matrix

- Correlations of PDF fit parameters (II)

	$\alpha_s(\mu_0)$	$\gamma_{1,\Delta}$	$\gamma_{1,us}$	$\gamma_{1,d}$	$\gamma_{2,d}$	A_s	b_s	a_s	$\gamma_{3,u}$	$m_c(m_c)$	$\gamma_{3,us}$	$m_b(m_b)$	a_Δ
a_u	-0.0435	0.0000	-0.8480	0.6008	0.1535	-0.0034	-0.0437	-0.0355	0.8111	0.0796	-0.4797	0.0044	-0.1718
b_u	-0.1251	0.0316	-0.8375	0.5537	0.1806	0.0008	-0.0345	-0.0276	0.7001	0.0625	-0.4889	-0.0005	-0.1452
$\gamma_{1,u}$	-0.0849	-0.0637	-0.8133	0.5422	0.1667	-0.0324	-0.0671	-0.0638	0.8948	0.0726	-0.4033	0.0075	-0.2028
$\gamma_{2,u}$	0.0920	0.1659	0.5760	-0.3308	-0.2276	0.0799	0.0966	0.1098	-0.9749	-0.0631	0.1728	-0.0142	0.2353
a_d	-0.0321	-0.0137	-0.7618	0.9630	-0.1842	0.0007	-0.0414	-0.0167	0.4878	0.0227	-0.4735	-0.0078	-0.2088
b_d	-0.1666	-0.1167	-0.6060	0.9351	-0.5969	-0.0064	-0.0249	-0.0203	0.3007	-0.0045	-0.3782	-0.0132	-0.2121
A_Δ	0.0206	0.8718	0.1649	-0.2544	0.1916	-0.0232	-0.0212	-0.0294	-0.2398	0.0202	0.0667	0.0034	0.9721
b_Δ	0.0086	-0.6291	-0.1067	-0.1834	-0.1103	0.0594	0.0577	0.0711	0.0052	-0.0063	-0.1768	-0.0083	-0.0662
A_{us}	0.0043	-0.0481	-0.8662	0.5862	0.0768	-0.0341	-0.0659	-0.0493	0.4485	0.1559	-0.8164	-0.0008	-0.0417
a_{us}	-0.0459	-0.0650	-0.8255	0.5493	0.0606	-0.0119	-0.0441	-0.0255	0.3870	0.0940	-0.8628	-0.0055	-0.0375
b_{us}	-0.0382	0.3783	0.7032	-0.3288	0.1278	-0.0734	-0.0445	-0.0807	-0.4262	-0.0100	0.3911	0.0040	0.1782
a_g	0.3785	0.0061	0.3050	-0.3280	0.1338	0.0936	0.0718	0.1165	-0.1744	-0.0137	0.4886	0.0323	-0.0360
b_g	-0.6085	0.1017	-0.0873	0.2827	-0.2104	-0.0543	-0.0114	-0.1223	0.2973	0.1560	-0.1337	0.0141	0.0066
$\gamma_{1,g}$	-0.4642	0.1021	-0.1778	0.3605	-0.1962	-0.0708	-0.0396	-0.1230	0.3132	0.0425	-0.1977	0.0071	0.0201

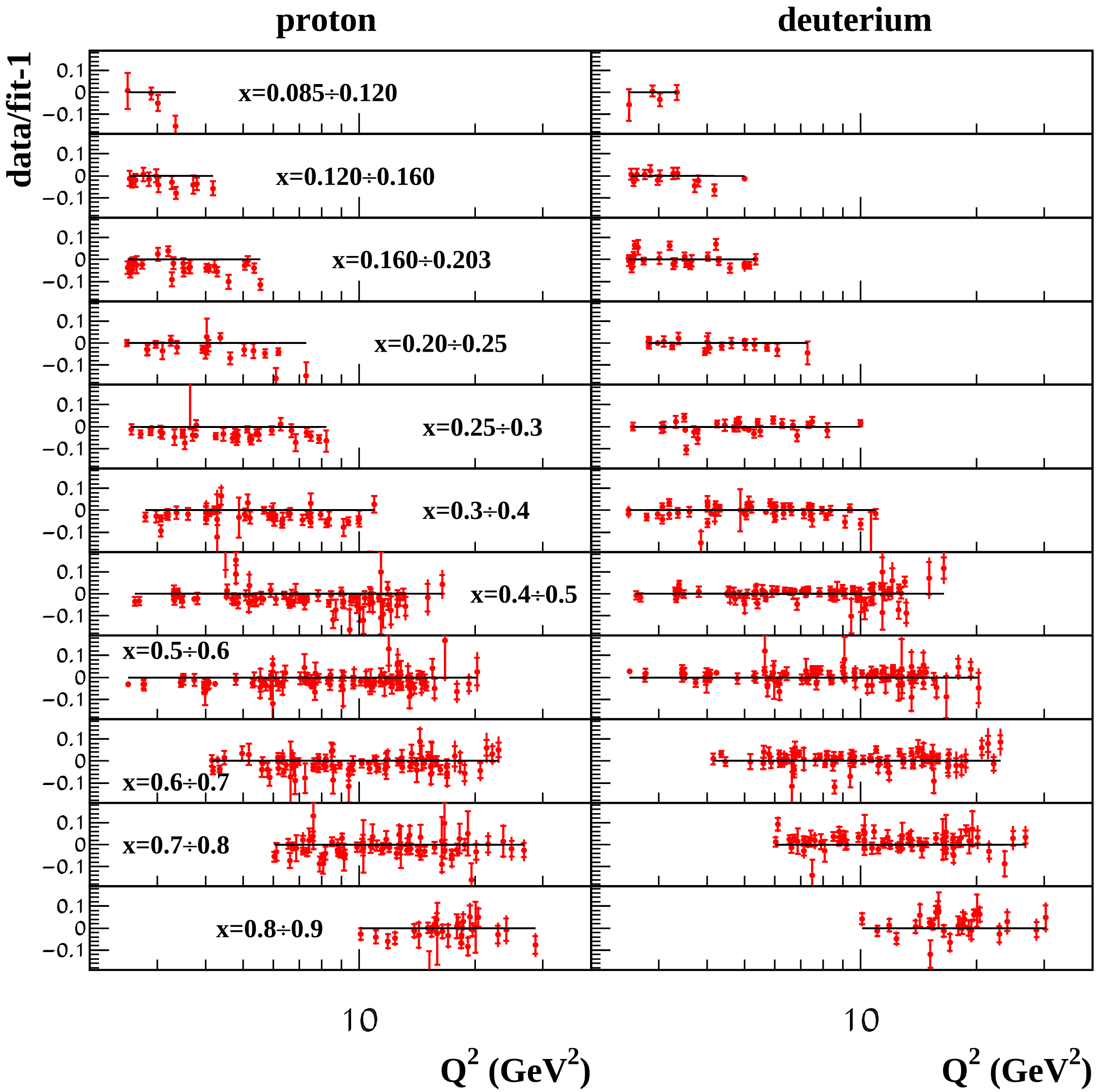
Covariance matrix

- Correlations of PDF fit parameters (III)

	$\alpha_s(\mu_0)$	$\gamma_{1,\Delta}$	$\gamma_{1,us}$	$\gamma_{1,d}$	$\gamma_{2,d}$	A_s	b_s	a_s	$\gamma_{3,u}$	$m_c(m_c)$	$\gamma_{3,us}$	$m_b(m_b)$	a_Δ
$\alpha_s(\mu_0)$	1.0000	0.0176	-0.0394	-0.0798	0.2357	-0.0018	-0.0982	-0.0075	-0.0291	0.1904	0.0676	0.0562	0.0136
$\gamma_{1,\Delta}$		1.0000	0.1183	-0.0802	0.2640	-0.0427	-0.0489	-0.0550	-0.1595	0.0193	0.0985	0.0069	0.7657
$\gamma_{1,us}$			1.0000	-0.6753	-0.0493	-0.0525	0.0158	-0.0445	-0.6039	-0.0656	0.6590	0.0017	0.1487
$\gamma_{1,d}$				1.0000	-0.4041	-0.0213	-0.0513	-0.0366	0.4145	0.0148	-0.3931	-0.0086	-0.2284
$\gamma_{2,d}$					1.0000	0.0308	-0.0016	0.0326	0.1801	0.0276	-0.0510	0.0111	0.1212
A_s						1.0000	0.8570	0.9749	-0.0664	-0.0206	-0.4355	0.0017	-0.0139
b_s							1.0000	0.8730	-0.0894	-0.0706	-0.3708	0.0005	-0.0127
a_s								1.0000	-0.0967	-0.1234	-0.4403	-0.0050	-0.0172
$\gamma_{3,u}$									1.0000	0.0674	-0.2082	0.0153	-0.2378
$m_c(m_c)$										1.0000	-0.0010	0.0505	0.0141
$\gamma_{3,us}$											1.0000	0.0083	0.0276
$m_b(m_b)$												1.0000	0.0006
a_Δ													1.0000

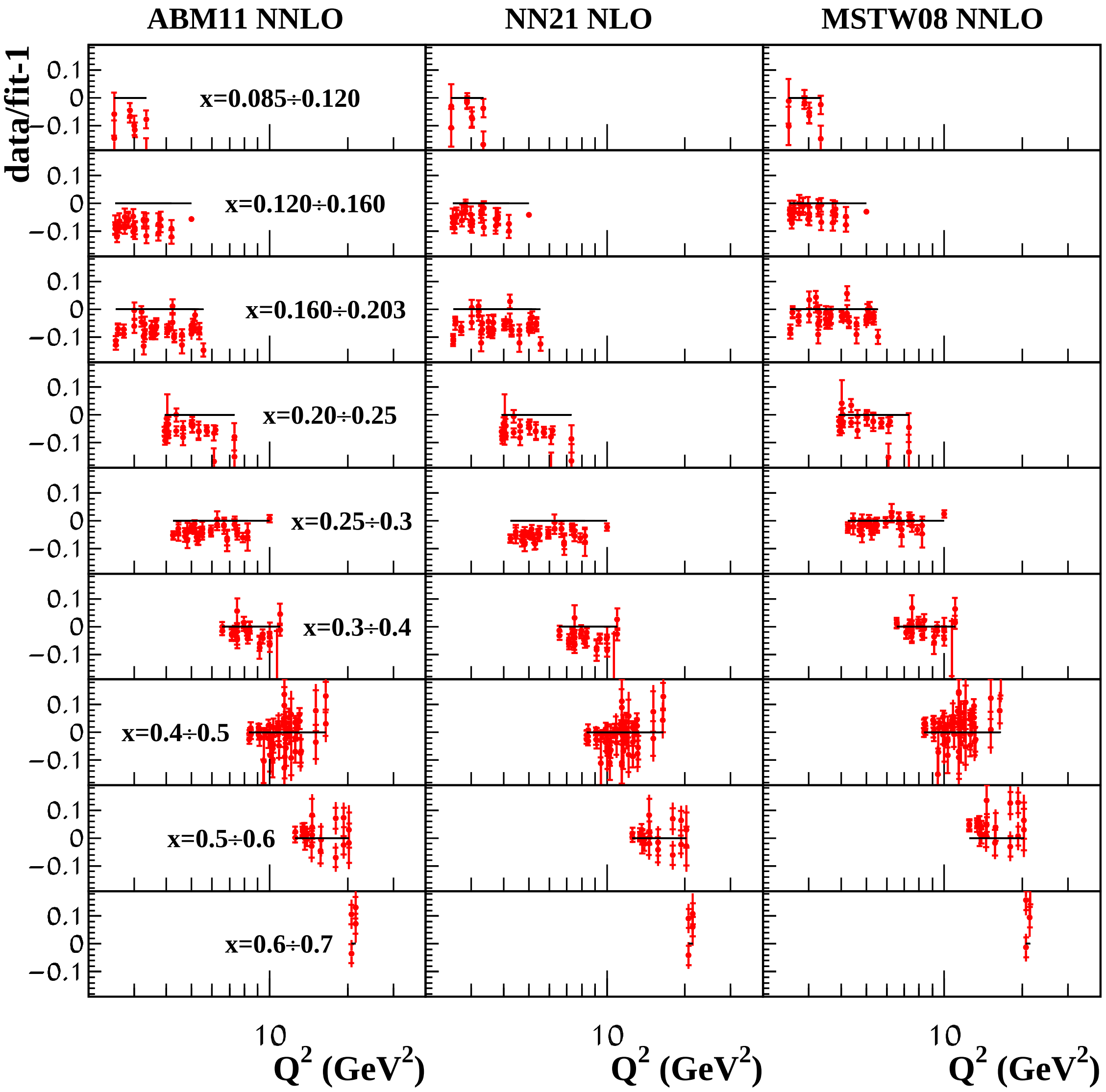
Pulls

- Comparision to SLAC inclusive DIS cross section data (proton and deuterium target)



Testing higher twist

- Fit of SLAC data without higher twist contributions (data cut at $W^2 > 12.5\ \mathrm{GeV}^2$, $Q^2 > 2.5\ \mathrm{GeV}^2$)



Strong coupling constant

Essential facts

- $\alpha_s(M_Z)$ from e^+e^- data high
- $\alpha_s(M_Z)$ from DIS data low
- World average 1992
  $\alpha_s(M_Z) = 0.117 \pm 0.004$

Bethke, Catani CERN TH-6484/92

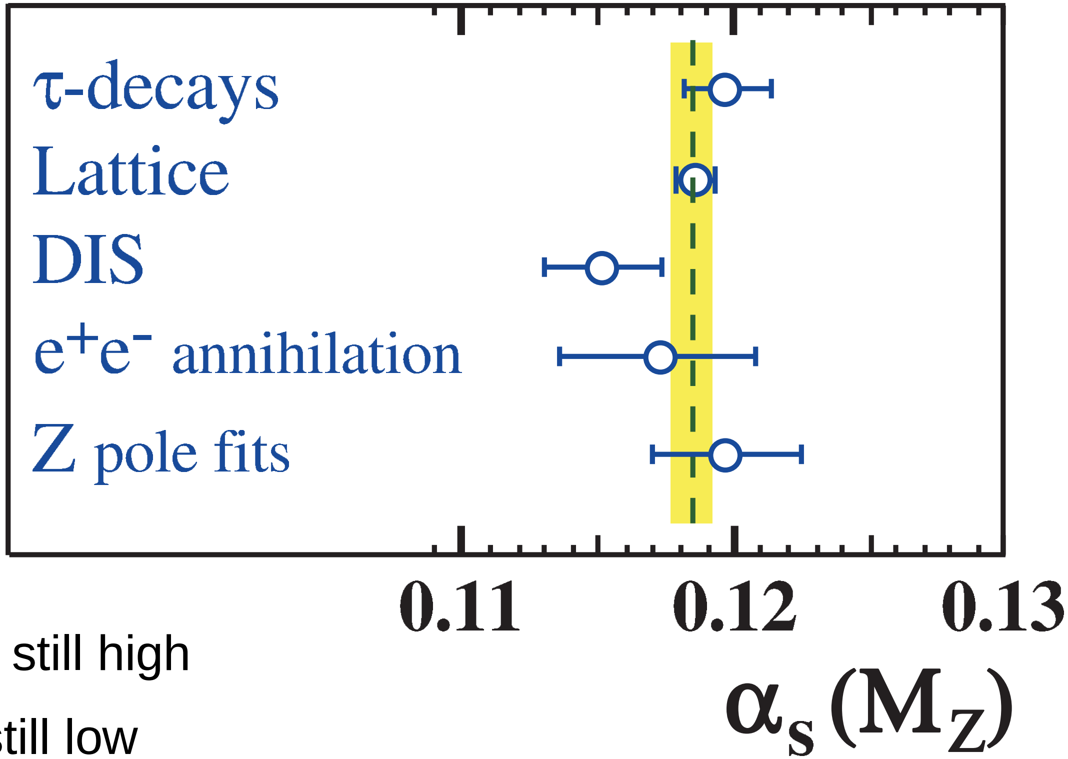
	Process	Ref.	Q [GeV]	$\alpha_s(Q)$	$\alpha_s(M_{Z^0})$	$\Delta\alpha_s(M_{Z^0})$ exp.	theor.	order of perturb.
1	R_τ [LEP]	[7-10]	1.78	$0.318\,^{+0.048}_{-0.039}$	$0.117\,^{+0.006}_{-0.005}$	$^{+0.003}_{-0.004}$	$^{+0.005}_{-0.004}$	NNLO
2	R_τ [world]	[2]	1.78	0.32 ± 0.04	$0.118\,^{+0.004}_{-0.006}$	–	–	NNLO
3	DIS [ν]	[3]	5.0	$0.193\,^{+0.019}_{-0.018}$	$0.111\,^{+0.006}_{-0.007}$	$^{+0.004}_{-0.006}$	0.004	NLO
4	DIS [μ]	[12]	7.1	0.180 ± 0.014	0.113 ± 0.005	0.003	0.004	NLO
5	$J/\Psi, \Upsilon$ decay	[4]	10.0	$0.167\,^{+0.015}_{-0.011}$	$0.113\,^{+0.007}_{-0.005}$	–	–	NLO
6	e^+e^- [σ_{had}]	[14]	34.0	0.163 ± 0.022	0.135 ± 0.015	–	–	NNLO
7	e^+e^- [shapes]	[15]	35.0	0.14 ± 0.02	0.119 ± 0.014	–	–	NLO
8	$p\bar{p} \to b\bar{b}X$	[11]	20.0	$0.136\,^{+0.025}_{-0.024}$	$0.108\,^{+0.015}_{-0.014}$	0.006	$^{+0.014}_{-0.013}$	NLO
9	$p\bar{p} \to W$ jets	[13]	80.6	0.123 ± 0.027	0.121 ± 0.026	0.018	0.020	NLO
10	$\Gamma(Z^0 \to \text{had.})$	[5]	91.2	0.133 ± 0.012	0.133 ± 0.012	0.012	$^{+0.003}_{-0.001}$	NNLO
11	Z^0 ev. shapes							
	ALEPH	[7]	91.2	$0.119\,^{+0.008}_{-0.010}$		–	–	NLO
	DELPHI	[8]	91.2	0.113 ± 0.007		0.002	0.007	NLO
	L3	[9]	91.2	0.118 ± 0.010		–	–	NLO
	OPAL	[10]	91.2	$0.122\,^{+0.006}_{-0.005}$		0.001	$^{+0.006}_{-0.005}$	NLO
	SLD	[6]	91.2	$0.120\,^{+0.015}_{-0.013}$		0.009	$^{+0.012}_{-0.009}$	NLO
	Average	[6-10]	91.2		0.119 ± 0.006	0.001	0.006	NLO
12	Z^0 ev. shapes							
	ALEPH	[7]	91.2	0.125 ± 0.005		0.002	0.004	resum.
	DELPHI	[8]	91.2	0.122 ± 0.006		0.002	0.006	resum.
	L3	[9]	91.2	0.126 ± 0.009		0.003	0.008	resum.
	OPAL	[10]	91.2	$0.122\,^{+0.003}_{-0.006}$		0.001	$^{+0.003}_{-0.006}$	resum.
	Average	[7-10]	91.2		0.123 ± 0.005	0.001	0.005	resum.

Table 1: Summary of measurements of α_s. For details see text.

2

α_s 2012

Bethke in PDG 2012



- $\alpha_s(M_Z)$ from e^+e^- data still high
- $\alpha_s(M_Z)$ from DIS data still low
- World average for $\alpha_s(M_Z)$ based on arithmetic average of (pre-averaged) $\alpha_s(M_Z)$ values from different methods/processes

Measurements of α_s

- Values of $\alpha_s(M_Z)$ at NNLO from PDF fits

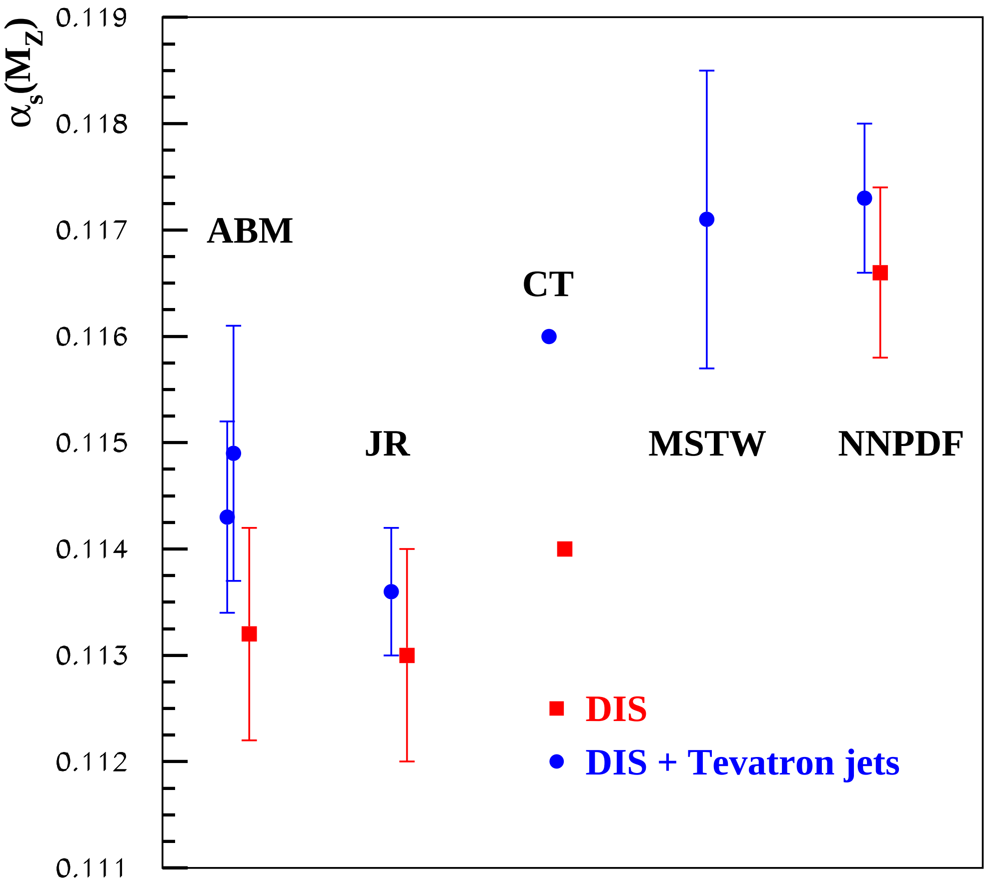
BBG	$0.1134\ {}^{+\ 0.0019}_{-\ 0.0021}$	valence analysis, NNLO	Blümlein, Böttcher, Guffanti '06
BB	0.1132 ± 0.0022	valence analysis, NNLO	Blümlein, Böttcher '12
GRS	0.112	valence analysis, NNLO	Glück, Reya, Schuck '06
ABKM	0.1135 ± 0.0014	HQ: FFNS $n_f = 3$	Alekhin, Blümlein, Klein, S.M. '09
ABKM	0.1129 ± 0.0014	HQ: BSMN-approach	Alekhin, Blümlein, Klein, S.M. '09
JR	0.1128 ± 0.0010	dynamical approach	Jimenez-Delgado, Reya '13
JR	0.1140 ± 0.0006	including jet data	Jimenez-Delgado, Reya '13
ABM11	0.1134 ± 0.0011		Alekhin, Blümlein, Klein, S.M. '11
ABM12	0.1133 ± 0.0011		Alekhin, Blümlein, Klein, S.M. '12
ABM12	0.1132 ± 0.0011	(without jets)	Alekhin, Blümlein, Klein, S.M. '12
MSTW	0.1171 ± 0.0014		Martin, Stirling, Thorne, Watt '09
NN21	0.1173 ± 0.0007		NNPDF '11
CTEQ	$0.1159 ... 0.1162$		CTEQ '13
CTEQ	0.1140	(without jets)	CTEQ '13

Measurements of α_s

- Values of $\alpha_s(M_Z)$ at NNLO from related measurements and lattice

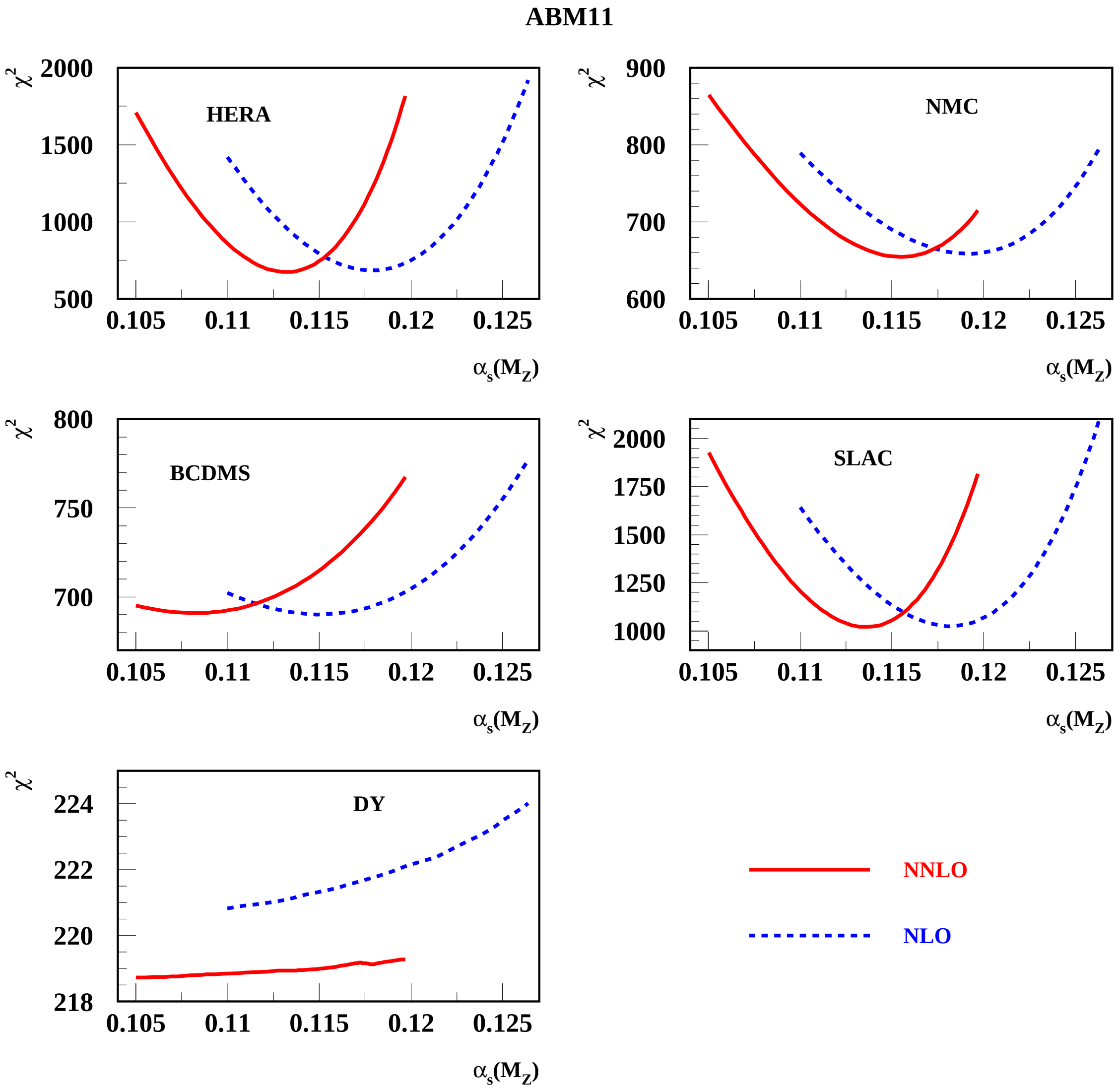
e^+e^- thrust	0.1140 ± 0.0015	Abbate et al.	arXiv:1204.5746
e^+e^- thrust	$0.1131 \, {}^{+\,0.0028}_{-\,0.0022}$	Gehrmann et al.	arXiv:1210.6945
3-jet rate	0.1175 ± 0.0025	Dissertori et al. 2009	arXiv:0910.4283
Z-decay	0.1189 ± 0.0026	BCK 2008/12 (N^{3}LO)	arXiv:0801.1821
			arXiv:1201.5804
τ decay	0.1212 ± 0.0019	BCK 2008	arXiv:0801.1821
τ decay	0.1204 ± 0.0016	Pich 2011	arXiv:1110.0016
τ decay	0.1191 ± 0.0022	Boito et al. 2012	arXiv:1203.3146
lattice	0.1205 ± 0.0010	PACS-CS 2009 (2+1 fl.)	arXiv:0906.3906
lattice	0.1184 ± 0.0006	HPQCD 2010	arXiv:1004.4285
lattice	0.1200 ± 0.0014	ETM 2012 (2+1+1 fl.)	arXiv:1201.5770
lattice	0.1156 ± 0.0022	Brambilla et al. 2012 (2+1 fl.)	arXiv:1205.6155
lattice	0.1181 ± 0.0014	JLQCD	arXiv:1002.0371
world average	0.1184 ± 0.0007	(2012)	arXiv:1210.0325

α_s *from DIS and PDFs*



- Significant spread of $\alpha_s(M_Z)$ values from DIS determinations
  Alekhin, Blümlein, S.M. '13

α_s *from DIS and PDFs*



- Profile of χ^2 for different data sets in ABM11 PDF fit Alekhin, Blümlein, S.M. '12

Comparison of α_s determinations

- Differences in α_s values:
  - result from different physics models and analysis procedures
    - target mass corrections (powers of nucleon mass M_N^2/Q^2)
    - higher twist $F_2^{\rm ht} = F_2 + ht^{(4)}(x)/Q^2 + ht^{(6)}(x)/Q^4 + \ldots$
    - error correlations
- Effects for differences between ABM, MSTW and NN21 understood
  - variants of ABM with no higher twist etc. reproduce larger α_s values

	α_s at NNLO	target mass corr.	higher twist	error correl.
ABM11	0.1134 ± 0.0011	**yes**	**yes**	**yes**
NNPDF21	0.1166 ± 0.0008	**yes**	**no**	**yes**
MSTW	0.1171 ± 0.0014	**no**	**no**	**no**

Treatment of heavy-quarks

Light quarks

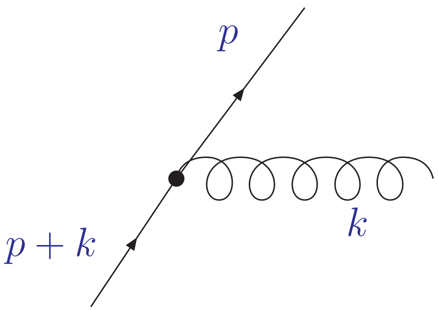
- Neglect “light quark” masses $m_u, m_d \ll \Lambda_{QCD}$ and $m_s < \Lambda_{QCD}$ in hard scattering process
  - scale-dependent u, d, s, g PDFs from mass singularities

Heavy quarks

- No mass singularities for $m_c, m_b, m_t \gg \Lambda_{QCD}$, no (evolving) PDFs
  - c and b PDFs for $Q \ggg m_c, m_b$ generated perturbatively
  - matching of two distinct theories
    $\longrightarrow n_f$ light flavors + heavy quark of mass m at low scales
    $\longrightarrow n_f + 1$ light flavors at high scales

Soft and collinear singularities

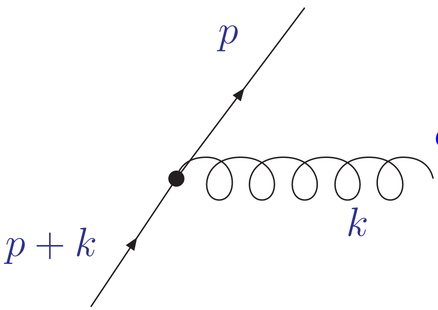
- Soft/collinear regions of phase space
  - massless partons



$$\frac{1}{(p+k)^2} \quad = \quad \frac{1}{2p \cdot k} = \frac{1}{2E_q E_g (1 - \cos\theta_{qg})}$$

Soft and collinear singularities

- Soft/collinear regions of phase space
  - massless partons



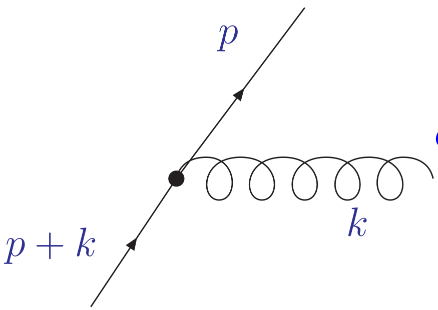
$$\frac{1}{(p+k)^2} = \frac{1}{2p \cdot k} = \frac{1}{2E_q E_g (1 - \cos\theta_{qg})}$$

$$\alpha_s \int d^4k \frac{1}{(p+k)^2} \longrightarrow \alpha_s \int dE_g \, d\theta_{qg} \, \frac{1}{2E_q E_g (1 - \cos\theta_{qg})}$$

$$\longrightarrow \alpha_s \frac{1}{\epsilon^2} \times (\ldots)$$ in dim. reg. $D = 4 - 2\epsilon$

Soft and collinear singularities

- Soft/collinear regions of phase space
  - massless partons

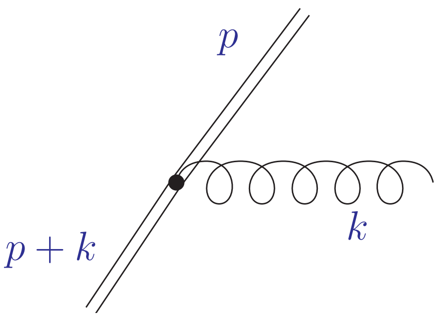


$$\frac{1}{(p+k)^2} = \frac{1}{2p\cdot k} = \frac{1}{2E_q E_g (1-\cos\theta_{qg})}$$

$$\alpha_s \int d^4k \frac{1}{(p+k)^2} \longrightarrow \alpha_s \int dE_g\, d\theta_{qg} \frac{1}{2E_q E_g (1-\cos\theta_{qg})}$$

$$\longrightarrow \alpha_s \frac{1}{\epsilon^2} \times (\ldots)$$ in dim. reg. $D = 4 - 2\epsilon$

- Parton masses regulate collinear singularity

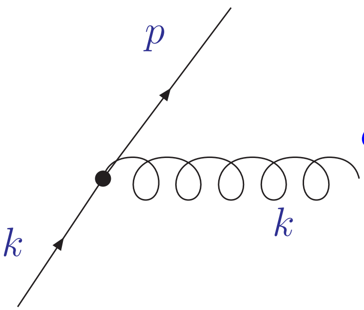


$$\frac{1}{(p+k)^2 - m_q^2} = \frac{1}{2p\cdot k} = \frac{1}{2E_q E_g (1-\beta\cos\theta_{qg})}$$

with $\beta = \left(1 - \frac{m_q^2}{E_q^2}\right)^{1/2} < 1$

Soft and collinear singularities

- Soft/collinear regions of phase space
  - massless partons

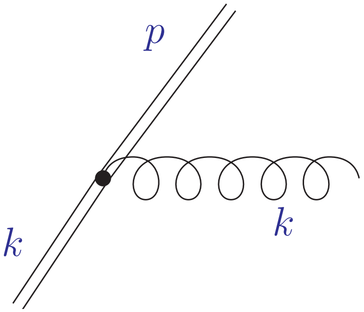


$$\frac{1}{(p+k)^2} = \frac{1}{2p\cdot k} = \frac{1}{2E_q E_g(1-\cos\theta_{qg})}$$

$$\alpha_s \int d^4k \frac{1}{(p+k)^2} \longrightarrow \alpha_s \int dE_g\, d\theta_{qg}\, \frac{1}{2E_q E_g(1-\cos\theta_{qg})}$$

$$\longrightarrow \alpha_s \frac{1}{\epsilon^2} \times (\dots)$$ in dim. reg. $D = 4 - 2\epsilon$

- Parton masses regulate collinear singularity



$$\frac{1}{(p+k)^2 - m_q^2} = \frac{1}{2p\cdot k} = \frac{1}{2E_q E_g(1-\beta\cos\theta_{qg})}$$

with $\beta = \left(1 - \frac{m_q^2}{E_q^2}\right)^{1/2} < 1$

$$\alpha_s \int d^4k \frac{1}{(p+k)^2 - m_q^2} \longrightarrow \alpha_s \frac{1}{\epsilon} \ln(m_q^2) \times (\dots)$$

Treatment of heavy-quarks

Charm structure function

- F_2^c at HERA (assume no "intrinsic charm")
  - $Q \not\gg m_c$: Fixed flavor-number scheme FFNS
    u, d, s, g partons and massive charm coeff. fcts.
  - $Q \ggg m_c$: Zero-mass variable flavor-number scheme ZM-VFNS
    terms $m_c/Q \to 0$, $n_f = 4$ PDFs (matching), $m_c = 0$ coeff. fcts.
  - $Q \gg m_c$: General-mass variable flavor-number scheme GM-VFNS
    terms $m_c/Q \neq 0$, but quasi-collinear logs $\ln(Q/m_c)$ large
    $n_f = 4$ PDFs, "interpolating" coeff. fcts. (matching prescriptions)

FFNS

- Perturbative QCD predictions for F_2^c and F_L^c (neutral current)
  - complete NLO predictions Laenen, Riemersma, Smith, van Neerven ‘92
  - approximate expressions to NNLO
    Laenen, S.M. ‘98; Alekhin, S.M. ‘08; Lo Presti, Kawamura, S.M., Vogt ‘10
  - asymptotic NNLO terms at large $Q^2 \gg m^2$ Bierenbaum, Blümlein, Klein ‘09

VFNS

- Variable flavor number schemes $\longrightarrow$ matching of two distinct theories
  Aivazis, Collins, Olness, Tung ‘94; Thorne, Roberts ‘98;
  Buza, Matiounine, Smith, van Neerven ‘98
  $\longrightarrow n_f$ light flavors + heavy quark of mass m at low scales
  $\longrightarrow n_f + 1$ light flavors at high scales
- Important aspects of variable flavor number schemes
  - mass factorization to be carried out before resummation
    $\longrightarrow$ mass factorization involves both heavy and light component of structure function
  - matching conditions required through NNLO
    Chuvakin, Smith, van Neerven ‘00
- Details of implementation matter in global fits

VFNS implementation

- GM-VFNS implementation using BSMN
  Buza, Matiounine, Smith, van Neerven '98
- DIS structure function F_2^h for heavy-quark h

  $F_2^{h,\mathrm{BMSN}}(N_f+1,x,Q^2) =$

  $$= \quad F_2^{h,\mathrm{exact}}(N_f,x,Q^2) + F_2^{h,\mathrm{ZMVFN}}(N_f+1,x,Q^2) - F_2^{h,\mathrm{asymp}}(N_f,x,Q^2)$$

  - $F_2^{h,\mathrm{exact}}$: massive heavy-quark structure function ($m \neq 0$)
  - $F_2^{h,\mathrm{ZMVFN}}$: DIS structure function with zero mass ($m = 0$)
  - $F_2^{h,\mathrm{asymp}}$: asymptotic expansion of heavy-quark structure function (logarithms $\ln(Q^2/m^2)$)

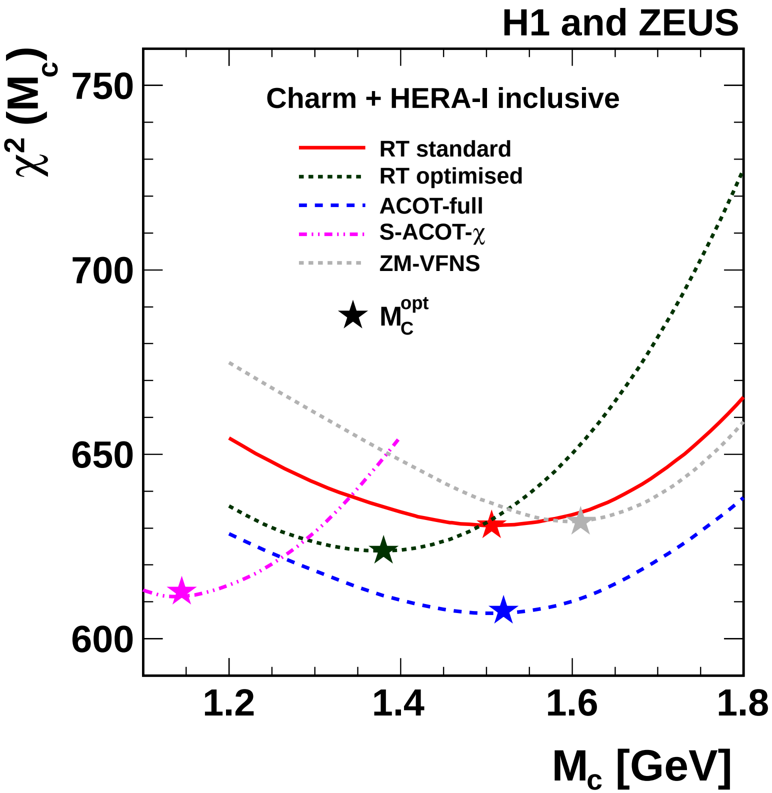
Heavy quark mass

- Data on F_2^c at HERA has correlation of m_c, $\alpha_S(M_Z)$, gluon PDF

  $\sigma_{c\bar{c}} \sim \alpha_s \, m_c^2 \, g(x)$

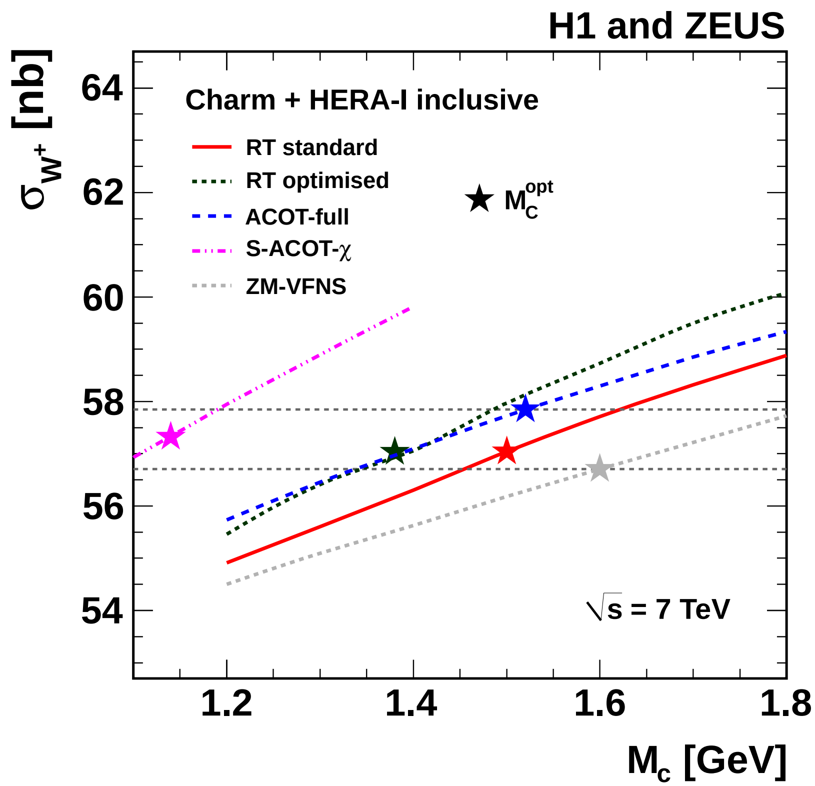
- Comparison of measured data with predictions in various VFNS schemes
  - data shows very good sensitivity to value of m_c
  - fit of value of m_c strongly dependent on particular choice of VFNS

    H1 coll. arxiv:1211.1182



Heavy quark mass

- Significant impact on cross section predictions at LHC
  - e.g., W^+-production



Quark masses in PDF fits

- Choice of value for heavy-quark masses part of uncertainty
- PDF fits assume pole mass scheme for heavy-quarks
  - numerical values systematically lower than those from PDG (2-loop conversion to pole mass)

[GeV]	PDG	ABKM	GJR	HERAPDF	MSTW	CT10	NNPDF21
m_c	1.66 $^{+0.09}_{-0.15}$	1.5 $^{+0.25}_{-0.25}$	1.3	1.4 $^{+0.25}_{-0.05}$	1.3	1.3	1.41
m_b	4.79 $^{+0.19}_{-0.08}$	4.5 $^{+0.5}_{-0.5}$	4.2	4.75 $^{+0.25}_{-0.45}$	4.75	4.75	4.75

PDG

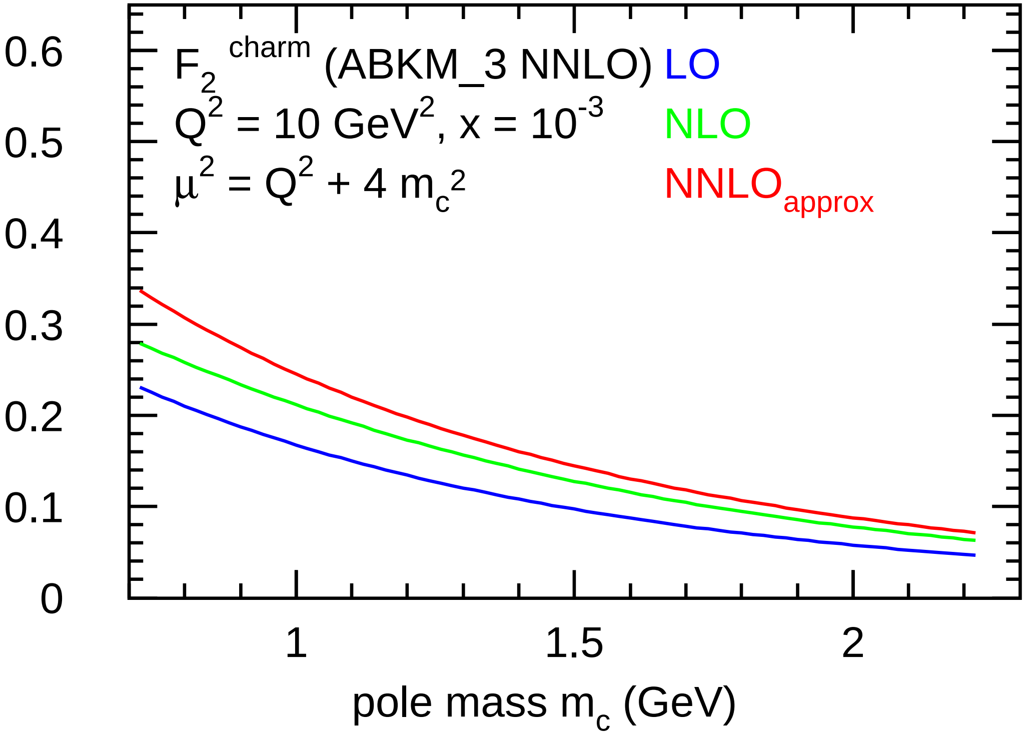
- PDG quotes running masses:
  charm: $m_c(m_c) = 1.27^{+0.07}_{-0.11}$ GeV, bottom: $m_b(m_b) = 4.20^{+0.17}_{-0.07}$ GeV

ABM11

- ABM11 uses running masses:
  charm: $m_c(m_c) = 1.27^{+0.08}_{-0.08}$ GeV, bottom: $m_b(m_b) = 4.19^{+0.13}_{-0.13}$ GeV

Running quark masses in DIS

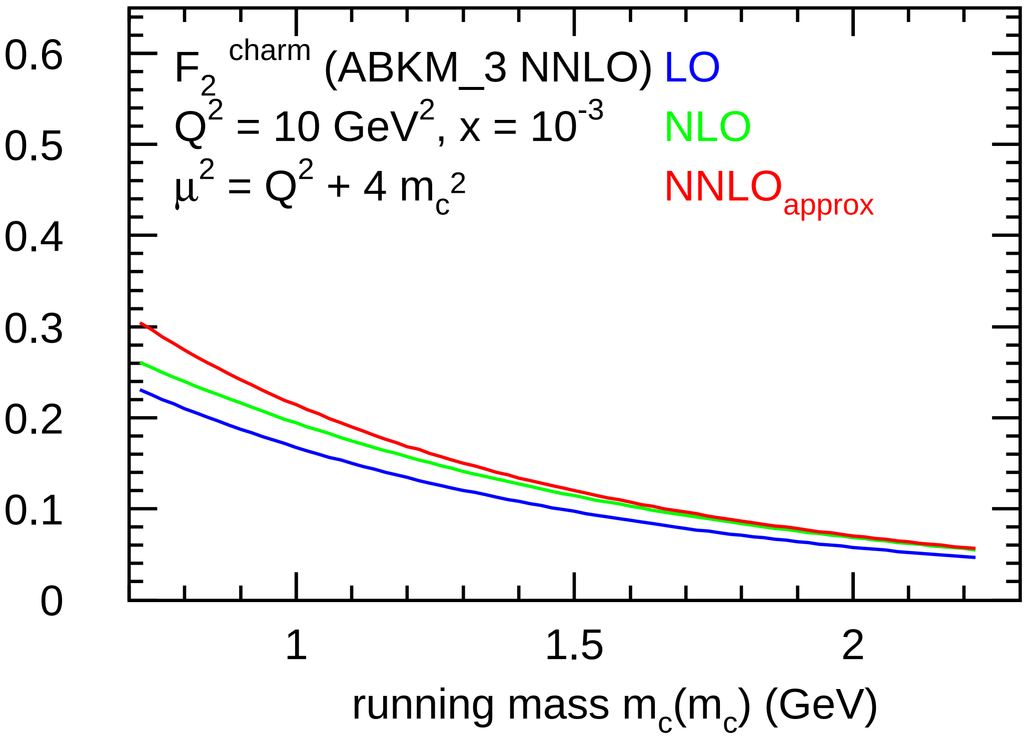
- Charm structure function



- Running quark masses in DIS
  - improved convergence
  - reduced scale dependence
- Comparison with pole mass scheme

Running quark masses in DIS

- Charm structure function



- Running mass
- Direct determination of $m_c(m_c)$ with all correlations

  Alekhin, Blümlein, Daum, Lipka, S.M. '12

  NLO

  $1.15 \pm 0.04\ (\mathrm{exp.})\ ^{+0.04}_{-0.00} (\mathrm{th.})$ GeV

  NNLO$_{\mathrm{approx}}$

  $1.24 \pm 0.03\ (\mathrm{exp.})\ ^{+0.03}_{-0.02} (\mathrm{th.})$ GeV

- PDG quotes running masses:

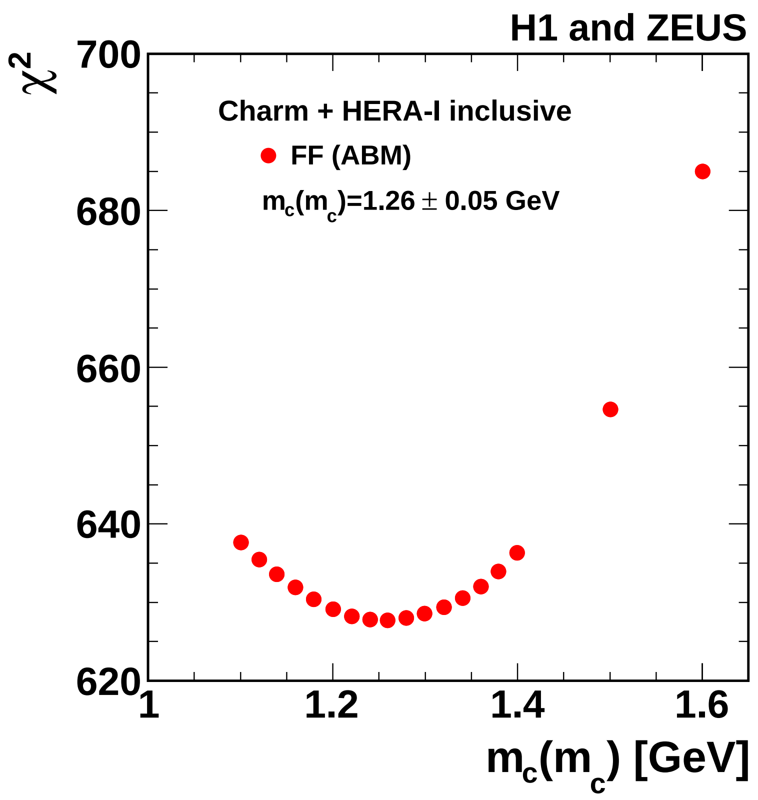
  $m_c(m_c) = 1.27^{+0.07}_{-0.11}$ GeV

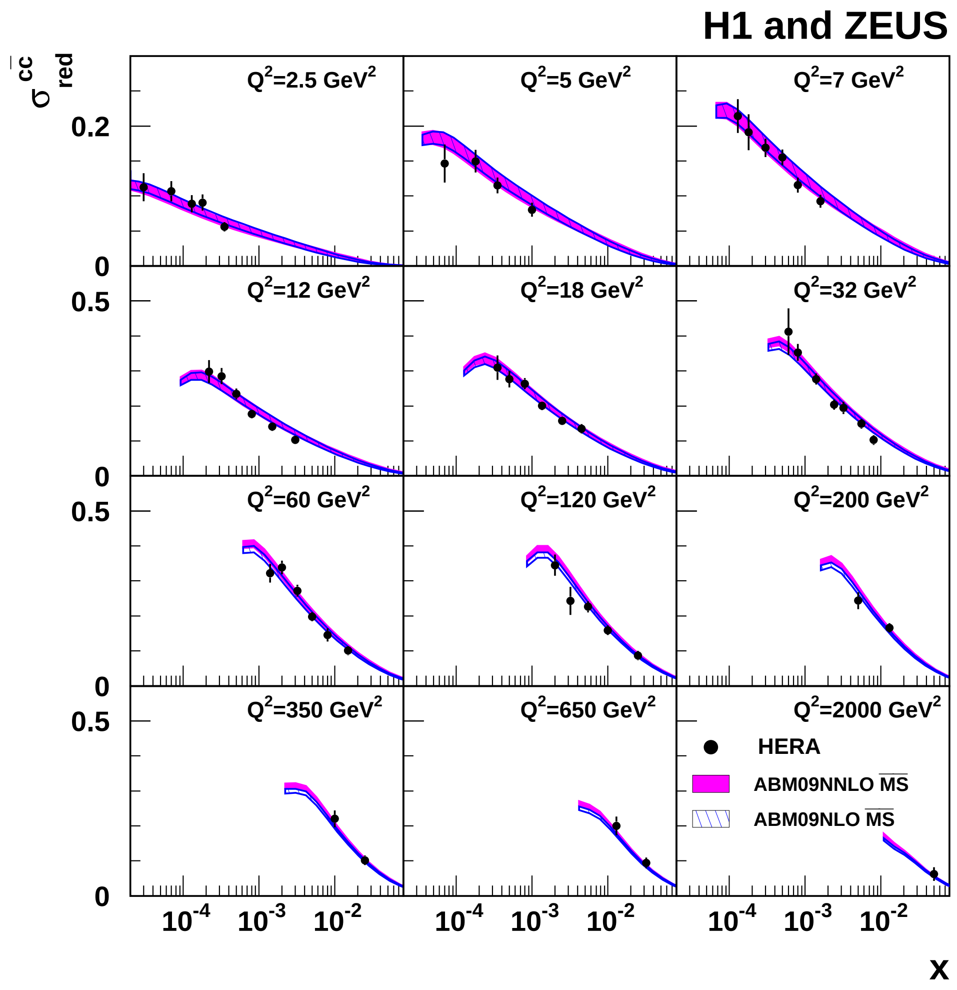
- Implicit $\alpha_s(M_Z)$ dependence in $m_c(m_c)$ determination from QCD sum rules

  Dehnadi, Hoang, Mateu, Zebarjad '11

Charm mass from HERA

- Determination of $\overline{\mathrm{MS}}$-mass $m_c(m_c)$ in DIS H1 coll. arxiv:1211.1182
- Very good description of data





LHC measurements

General remarks

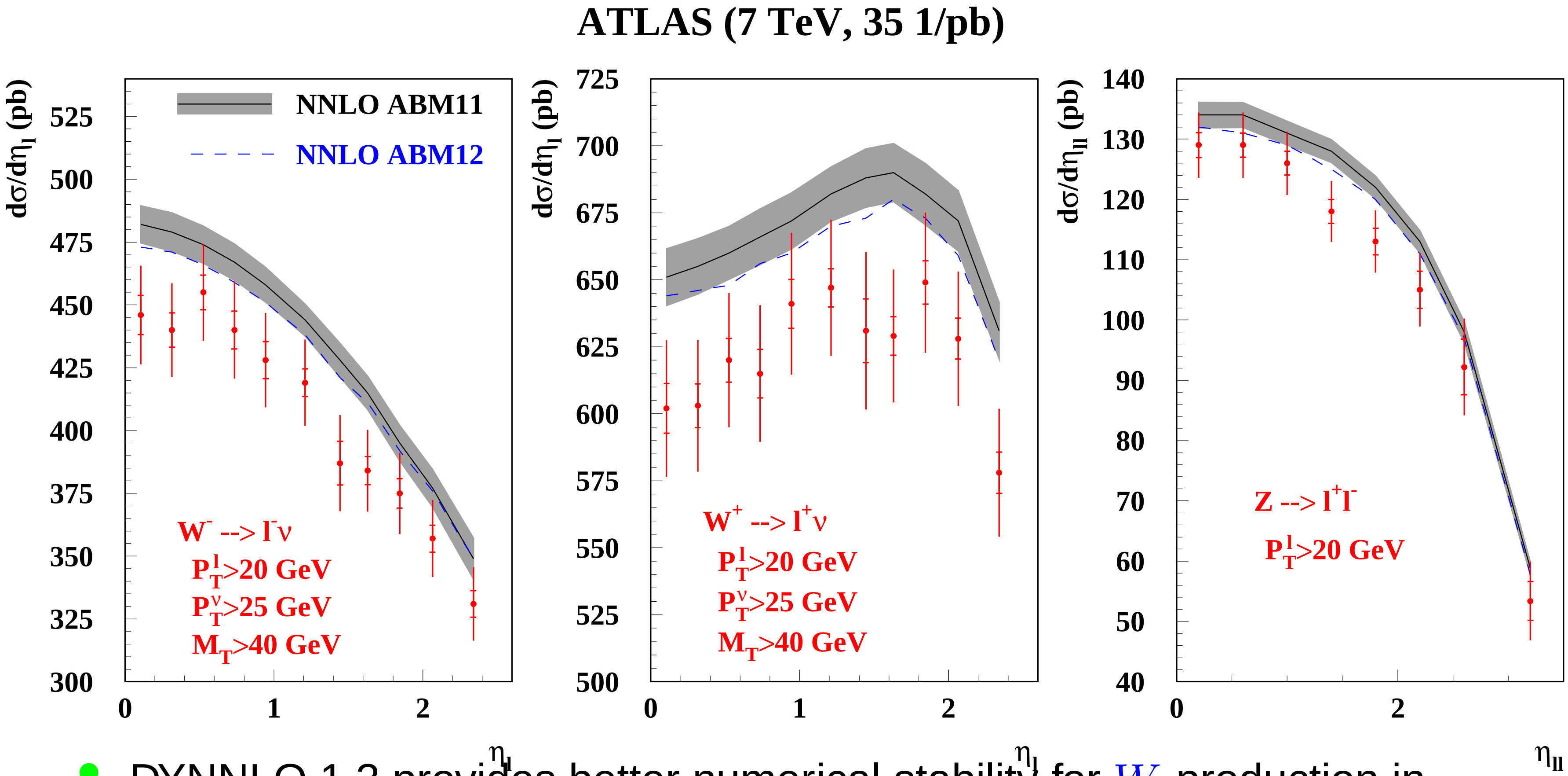
- QCD corrections important
  - require theory predictions to NNLO accuracy
- PDF fits with 3-flavors for DIS, 5-flavors for LHC data (matching from 3 to 5-flavors)
  - QCD evolution over large range

Benchmark processes

- Complete NNLO QCD corrections available for
  - $W^{\pm}$- and Z-boson production
    Hamberg, van Neerven, Matsuura '91; Harlander, Kilgore '02
  - top-quark hadro-production Czakon, Fiedler, Mitov '13
- Jet data from Tevatron and LHC
  - QCD corrections only NLO known
  - possible impact of jet definition and algorithm
  - ongoing effort towards NNLO
    Gehrmann-De Ridder, Gehrmann, Glover, Pires '13

ABM PDFs with LHC data

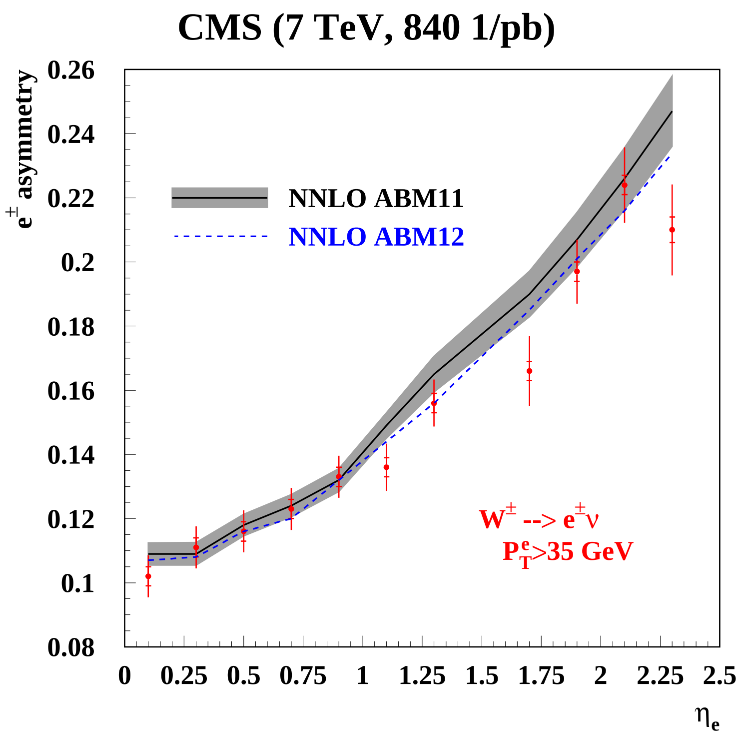
Fit to LHC Drell-Yan data Alekhin, Bümlein, S.M. '13

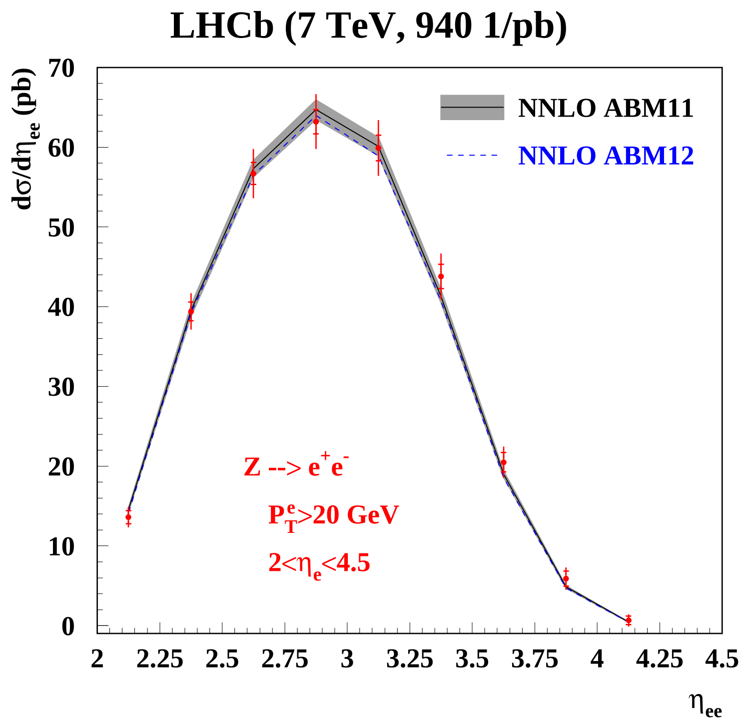


- DYNNLO 1.3 provides better numerical stability for W-production in central region ($\sim$ 200h) Catani, Cieri, Ferrera, de Florian, Grazzini '09
- FEWZ 3.1 more convenient/stable for estimation of PDF uncertainties ($\sim$ 2d x 24 processors) Li, Petriello '12
- Central values computed with DYNNLO and the PDF errors with FEWZ

ABM PDFs with LHC data

Fit to LHC Drell-Yan data Alekhin, Bümlein, S.M. ‘13

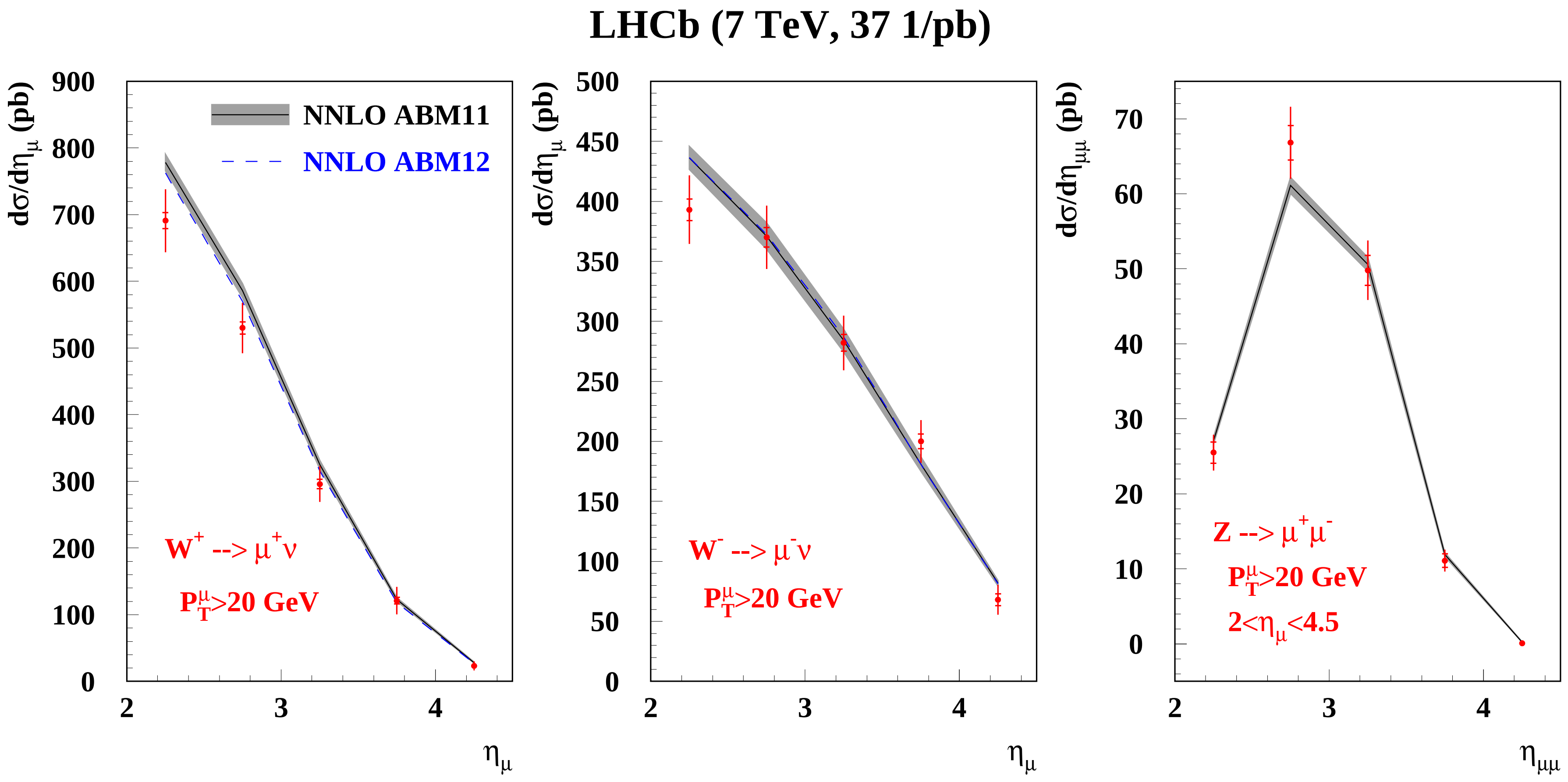




- Good overall agreement with data of CMS ‘10 and LHCb ‘12, ‘13

ABM PDFs with LHC data

Fit to LHC Drell-Yan data Alekhin, Bümlein, S.M. '13



- Good overall agreement with data of CMS '10 and LHCb '12, '13

Benchmarking of ABM PDFs

Experiment	ATLAS '11	CMS '12	LHCb 12	LHCb '12
Final states	$W^+ \to l^+\nu$ $W^- \to l^-\nu$ $Z \to l^+l^-$	$W^+ \to e^+\nu$ $W^- \to e^-\nu$	$W^+ \to \mu^+\nu$ $W^- \to \mu^-\nu$	$Z \to e^+e^-$
Luminosity (1/pb)	35	840	37	940
NDP	30	11	10	9
χ^2 (ABM11)	35.7(7.7)	10.6(4.7)	13.1(4.5)	11.3(4.2)
χ^2 (ABM12)	31.5	10.8	15.2	10.3
χ^2 (ABM12)/part.	32.2	10.9	13.0	8.7

- value of χ^2 for Drell-Yan data at the LHC with NNLO ABM11 PDFs (+ one standard deviation of χ^2 equal to $\sqrt{2NDP}$)
- ABM11 benchmarking in arXiv:1211.5142 reports wrong χ^2 values for PDF comparison (NLO MCFM with K-factors, no PDF errors, shifted α_s)

Theory predictions

Drell-Yan process

- $W^{\pm}$- and Z-boson production

  Hamberg, van Neerven, Matsuura '91; Harlander, Kilgore '02

  - theory (scale) + 1σ PDF uncertainty

LHC7	W^+	W^-	$W^\pm$	Z
ABM11	$59.53\ ^{+0.38}_{-0.23}\ ^{+0.88}_{-0.88}$	$39.97\ ^{+0.28}_{-0.17}\ ^{+0.65}_{-0.65}$	$99.51\ ^{+0.69}_{-0.41}\ ^{+1.43}_{-1.43}$	$29.23\ ^{+0.18}_{-0.10}\ ^{+0.42}_{-0.42}$
ABM12	$58.40\ ^{+0.38}_{-0.24}\ ^{+0.70}_{-0.70}$	$39.63\ ^{+0.29}_{-0.18}\ ^{+0.45}_{-0.45}$	$98.03\ ^{+0.67}_{-0.41}\ ^{+1.13}_{-1.13}$	$28.79\ ^{+0.17}_{-0.11}\ ^{+0.33}_{-0.33}$

Higgs production

- Gluon-gluon fusion at NNLO

  Harlander, Kilgore '02; Anastasiou, Melnikov '02; Ravindran, Smith, van Neerven '03

  - Higgs boson mass $m_H = 125\ \mathrm{GeV}$
  - theory (scale) + 1σ PDF uncertainty

	LHC7	LHC8	LHC13	LHC14
ABM11	$13.23\ ^{+1.35}_{-1.31}\ ^{+0.30}_{-0.30}$	$16.99\ ^{+1.69}_{-1.63}\ ^{+0.37}_{-0.37}$	$39.57\ ^{+3.60}_{-3.42}\ ^{+0.77}_{-0.77}$	$44.68\ ^{+4.02}_{-3.78}\ ^{+0.85}_{-0.85}$
ABM12	$13.28\ ^{+1.35}_{-1.32}\ ^{+0.31}_{-0.31}$	$17.05\ ^{+1.68}_{-1.64}\ ^{+0.39}_{-0.39}$	$39.69\ ^{+3.60}_{-3.42}\ ^{+0.84}_{-0.84}$	$44.81\ ^{+4.01}_{-3.80}\ ^{+0.94}_{-0.94}$

Summary

Parton distributions at the LHC

- Precision determinations of non-perturbative parameters is essential
  - parton content of proton (PDFs)
  - coupling constants $\alpha_s(M_Z)$
  - masses m_c, m_b, m_t, M_W, m_H, . . .
- Precision measurements require careful definition of observable
  - confronting LHC data requires continuous benchmarking
  - source of interesting observations
- Radiative corrections at higher orders in QCD and EW are mandatory
  - NNLO in QCD is *conditio sine qua non*
  - theory improvements driven by experimental precision
- Lots of challenging tasks for young researchers