

# *The Higgs-boson, the top-quark and electroweak vacuum stability*

**Sven-Olaf Moch**

*Universität Hamburg & DESY, Zeuthen*

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*LHCPhenoNet Summer School, Cracow, Sep 12, 2013*

# *Fate of the universe*

*Higgs boson mass* Atlas & CMS coll. '13

$$m_H = 125.6 \pm 0.3 \text{ GeV}$$

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*Bound on from vacuum stability*

$$m_H \geq 129.2 \text{ GeV}$$

# Fate of the universe

## Higgs boson too light ? Are we doomed ?

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**WORLD**

### Scientists studying so-called subatomic 'God particle' say there will be an universe-ending 'catastrophe'

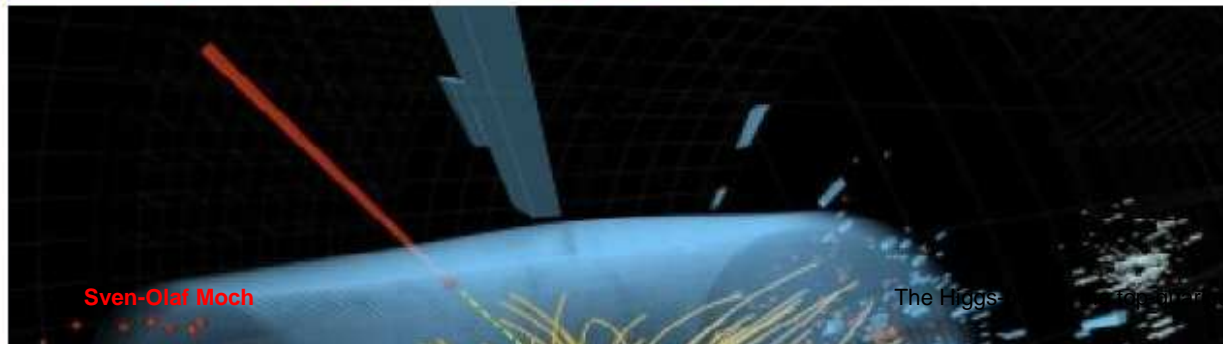
The end of the universe won't come for tens of billions of years, but when it does happen it will destroy everything, according to researchers studying the Higgs boson particle. "If you use all the physics that we know now and you do what you think is a straightforward calculation, it's bad news," theoretical physicist Joseph Lykken said Monday.

Comments (24)

REUTERS

TUESDAY, FEBRUARY 19, 2013, 11:44 AM

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# *Fate of the universe*

*Well, ...* [check the fine print]

$$m_H \geq 129.2\text{GeV} + 1.8 \times \left( \frac{m_t - 173.2\text{GeV}}{0.9\text{GeV}} \right) + \dots$$

# Top quark mass

*Experimental result* CDF & D0 coll. 1305.3929

$$m_t = 173.20 \pm 0.87 \text{ GeV}$$

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*Which is the value of the top quark mass ?*

$$m_t = ?$$

*Which top quark mass has this value ?*

$$? = 173.20 \pm 0.87 \text{ GeV}$$



# *Introduction*

## *Classical mechanics*

- Mass is defined as product of density and volume of matter
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  - classical concept
- *The quantity of matter is that which arises jointly from its density and magnitude.*

*A body twice as dense in double the space is quadruple in quantity. This quantity I designate by the name of body or of mass.*

Newton

### PHILOSOPHIÆ NATURALIS PRINCIPIA MATHEMATICA.

#### DEFINITIONES.

##### DEFINITIO I.

*Quantitas materiæ est mensura ejusdem orta ex illius densitate et magnitudine conjunctim.*

**A**ER densitate duplicata, in spatio etiam duplicato, fit quadruplus; in triplicato sextuplus. Idem intellige de nive & pulveribus per compressionem vel liquefactionem condensatis. Et par est ratio corporum omnium, quæ per causas quascunque diversimode condensantur. Medii interea, si quod fuerit, interstitia partium libere pervadentis, hic nullam rationem habeo. Hanc autem quantitatem sub nomine corporis vel massæ in sequentibus passim intelligo. Innotescit ea per corporis cujusque pondus: Nam ponderi proportionalem esse reperi per experimenta pendulorum accuratissime instituta, uti posthac docebitur.

##### DEFINITIO II.

*Quantitas motus est mensura ejusdem orta ex velocitate et quantitate materiæ conjunctim.*

Motus totius est summa motuum in partibus singulis; ideoque in corpore duplo majore, æquali cum velocitate, duplus est, & dupla cum velocitate quadruplus.

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- Mass is conserved Lavoisier
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$$M(X) = N_A m_a(X) \text{ Avogadro}$$

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## Special relativity

- Equivalence principle

$$E = mc^2 \text{ Einstein}$$

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# Standard Model

- Higgs boson gives mass to matter fields via Higgs-Yukawa coupling
  - large top quark mass  $m_t$

## QCD

- Classical part of QCD Lagrangian

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu}^a F_b^{\mu\nu} + \sum_{\text{flavors}} \bar{q}_i (i\not{D} - m_q)_{ij} q_j$$

- field strength tensor  $F_{\mu\nu}^a$  and matter fields  $q_i, \bar{q}_j$
- covariant derivative  $D_{\mu,ij} = \partial_\mu \delta_{ij} + ig_s (t_a)_{ij} A_\mu^a$
- Formal parameters of the theory (no observables)
  - strong coupling  $\alpha_s = g_s^2 / (4\pi)$
  - quark masses  $m_q$

## Challenge

- Suitable observables for measurements of  $\alpha_s, m_q, \dots$ 
  - comparison of theory predictions and experimental data

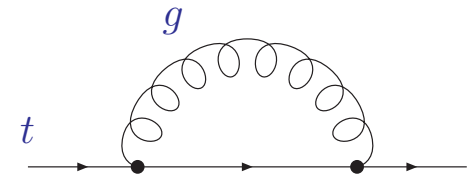
# Quark mass renormalization

- Heavy-quark self-energy  $\Sigma(p, m_q)$

$$\text{---} + \text{---} \circlearrowleft \Sigma \text{---} + \text{---} \circlearrowleft \Sigma \text{---} \circlearrowleft \Sigma \text{---} + \dots = \frac{i}{\not{p} - m_q - \Sigma(p, m_q)}$$

## QCD

- QCD corrections to self-energy  $\Sigma(p, m_q)$ 
  - dimensional regularization  $D = 4 - 2\epsilon$
  - one-loop: UV divergence  $1/\epsilon$  (Laurent expansion)



$$\Sigma^{(1), \text{bare}}(p, m_q) = \frac{\alpha_s}{4\pi} \left( \frac{\mu^2}{m_q^2} \right)^\epsilon \left\{ (\not{p} - m_q) \left( -C_F \frac{1}{\epsilon} + \text{fin.} \right) + m_q \left( 3C_F \frac{1}{\epsilon} + \text{fin.} \right) \right\}$$

- Relate bare and renormalized mass parameter  $m_q^{\text{bare}} = m_q^{\text{ren}} + \delta m_q$

$$\text{---} \circlearrowleft \Sigma^{\text{ren}} \text{---} = \text{---} + \text{---} \circlearrowleft \Sigma^{\text{bare}} \text{---} + \text{---} \times \text{---} + \dots$$

$$\Sigma^{\text{ren}}(p, m_q) \qquad (Z_\psi - 1)\not{p} - (Z_m - 1)m_q$$

# Mass renormalization scheme

## Pole mass

- Based on (unphysical) concept of top quark being a free parton
  - $m_q^{\text{ren}}$  coincides with pole of propagator at each order

$$\not{p} - m_q - \Sigma(p, m_q) \Big|_{\not{p}=m_q} \rightarrow \not{p} - m_q^{\text{pole}}$$

- Definition of pole mass ambiguous up to corrections  $\mathcal{O}(\Lambda_{QCD})$ 
  - heavy-quark self-energy  $\Sigma(p, m_q)$  receives contributions from regions of all loop momenta – also from momenta of  $\mathcal{O}(\Lambda_{QCD})$

## $\overline{MS}$ scheme

- $\overline{MS}$  mass definition
  - one-loop minimal subtraction

$$\delta m_q^{(1)} = m_q \frac{\alpha_s}{4\pi} 3C_F \left( \frac{1}{\epsilon} - \gamma_E + \ln 4\pi \right)$$

- $\overline{MS}$  scheme induces scale dependence:  $m(\mu)$

# *Scheme transformations*

- Conversion between different renormalization schemes possible in perturbation theory
- Relation for pole mass and  $\overline{MS}$  mass
  - known to three loops in QCD Gray, Broadhurst, Gräfe, Schilcher '90; Chetyrkin, Steinhauser '99; Melnikov, v. Ritbergen '99
  - example: one-loop QCD

$$m^{\text{pole}} = m(\mu) \left\{ 1 + \frac{\alpha_s(\mu)}{4\pi} \left( \frac{4}{3} + \ln \left( \frac{\mu^2}{m(\mu)^2} \right) \right) + \dots \right\}$$



# Running quark mass

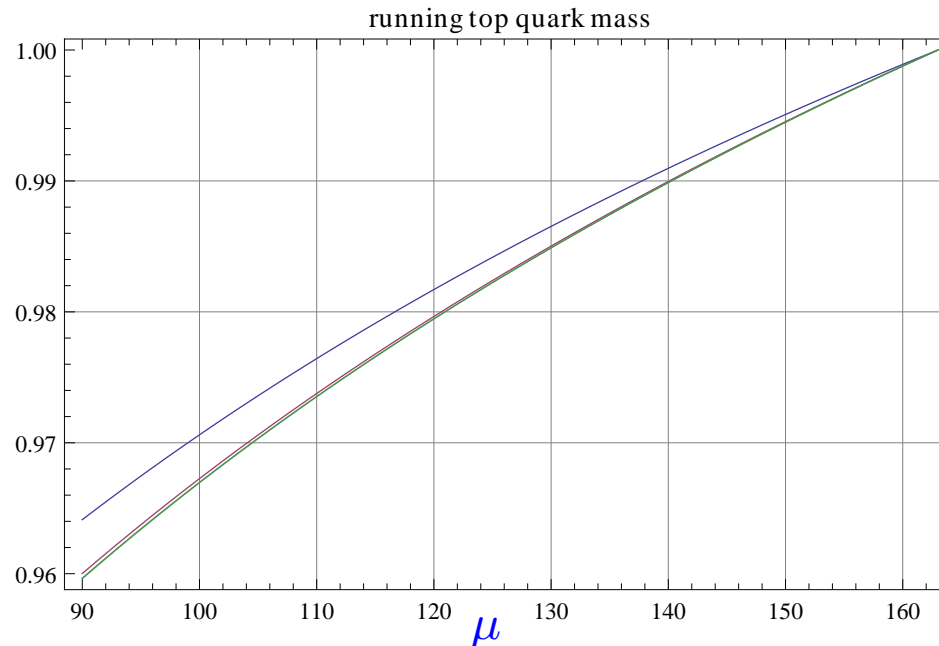
## Scale dependence

- Renormalization group equation for scale dependence
  - mass anomalous dimension  $\gamma$  known to four loops

Chetyrkin '97; Larin, van Ritbergen, Vermaseren '97

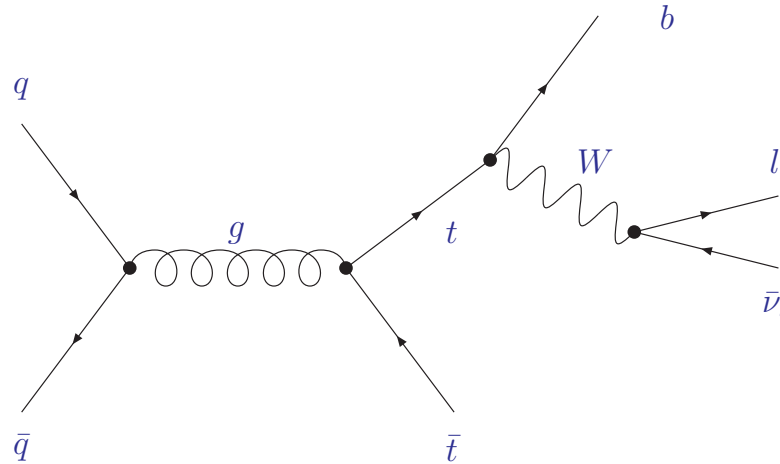
$$\left( \mu^2 \frac{\partial}{\partial \mu^2} + \beta(\alpha_s) \frac{\partial}{\partial \alpha_s} \right) m(\mu) = \gamma(\alpha_s) m(\mu)$$

- Plot mass ratio  $m_t(163\text{GeV})/m_t(\mu)$



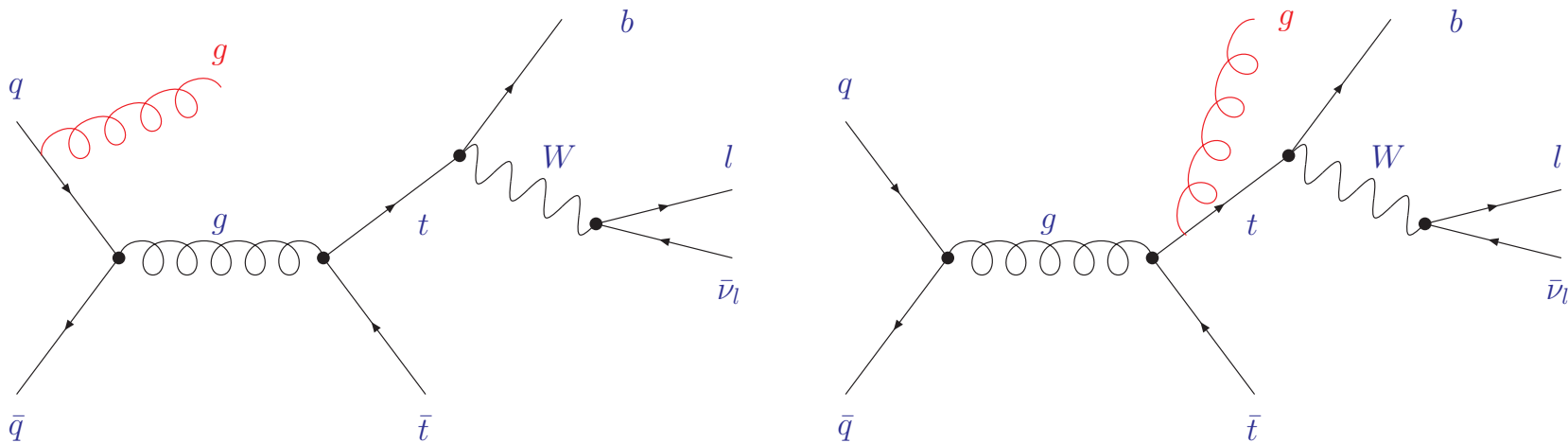
# Hard scattering process

- Born process ( $q\bar{q}$ -channel) with leptonic decay  $t \rightarrow b l \bar{\nu}_l$



# Radiative corrections

- Real corrections (examples): gluon emission
  - phase space integration  $\rightarrow$  infrared divergences (soft/collinear singularities)

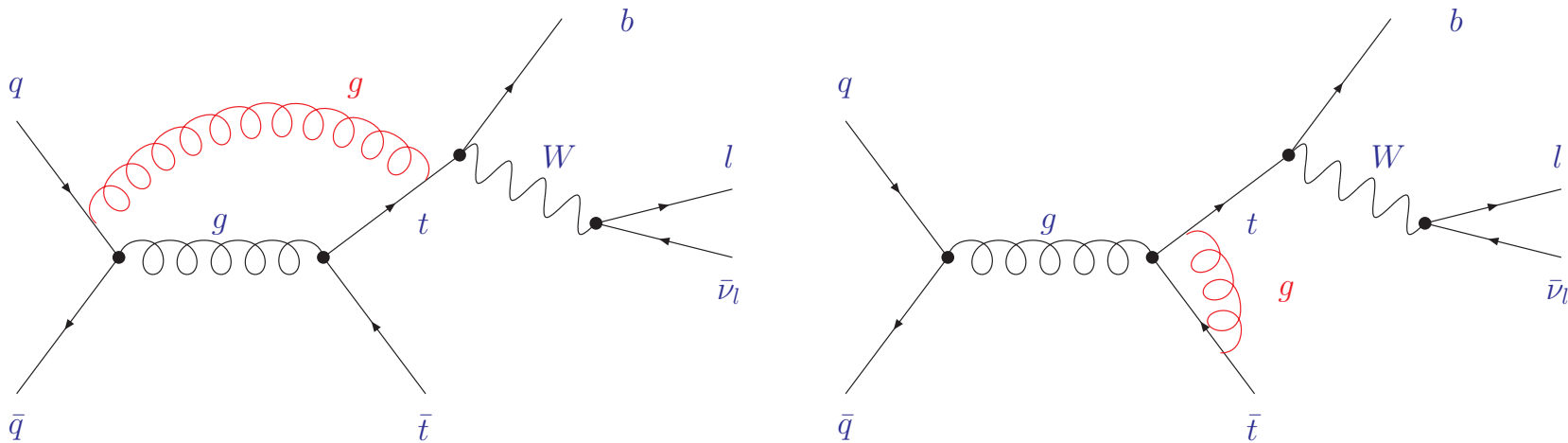


- Parton shower MC
  - emission probability modeled by Sudakov exponential with cut-off  $Q_0$
  - leading logarithmic accuracy

$$\Delta(Q^2, Q_0^2) = \exp\left(-C_F \frac{\alpha_s}{2\pi} \ln\left(\frac{Q^2}{Q_0^2}\right)\right)$$

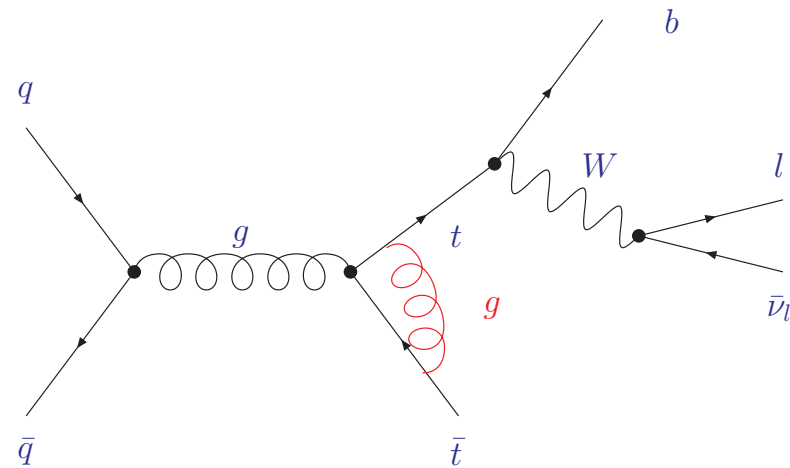
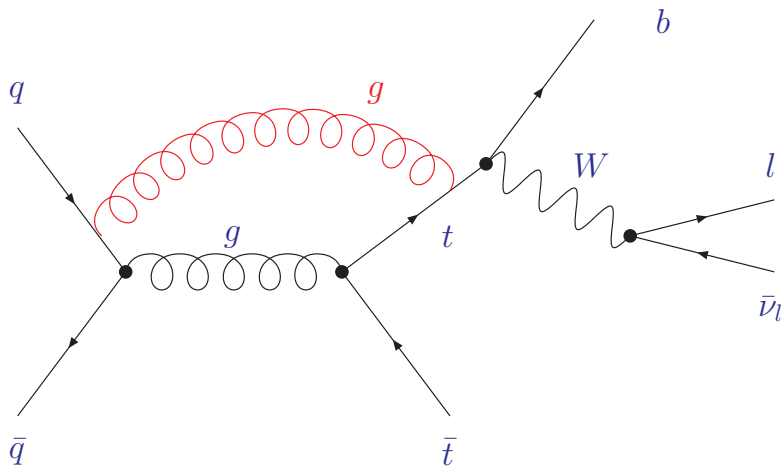
# Radiative corrections

- Virtual corrections (examples): gluon exchange
  - box diagram (left) and vertex corrections (right)
  - infrared divergences cancel against real emission contributions

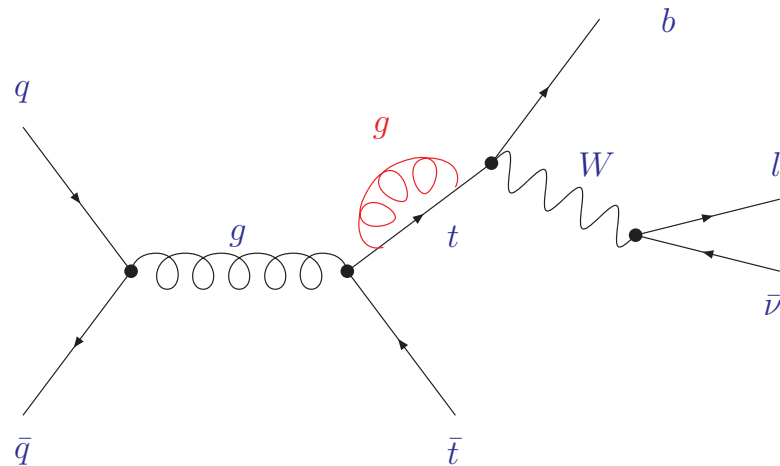


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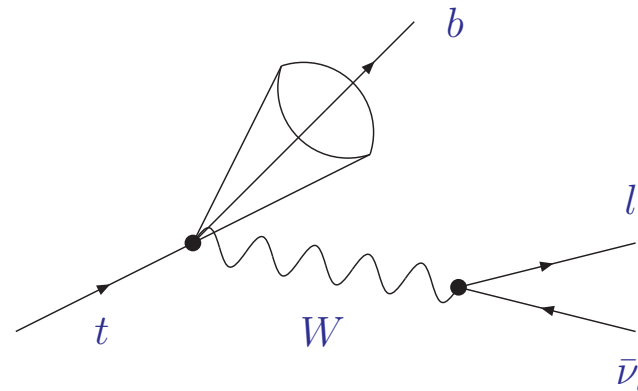


- Mass renormalization from self-energy corrections to top quark



# Current methods

- Current methods based on reconstructed physics objects
  - jets, identified charged leptons, missing transverse energy
  - $m_t^2 = (p_{W\text{-boson}} + p_{b\text{-jet}})^2$



## Template method

- Distributions of kinematically reconstructed top mass values compared to templates for nominal top mass values
  - distributions rely on parton shower predictions
  - no NLO corrections applied

## Matrix element method

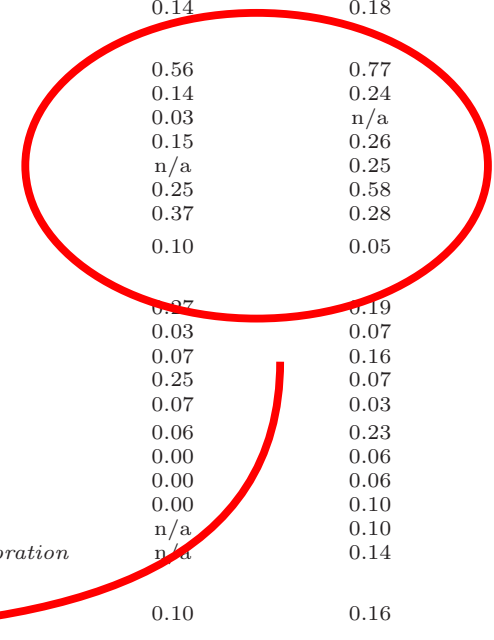
- Event-by-event likelihood for kinematic configurations arising from events of a given top mass.
  - tree level matrix elements only
  - combinatorics of assignment of jets to top quarks

# Tevatron combination

TABLE VIII: Individual components of uncertainty on CDF and D0  $m_t$  measurements in the lepton+jets channel for Run II data [26, 27].

Systematic Source	Uncertainty [GeV]	
	CDF (5.6 fb <sup>-1</sup> ) $m_t = 173.00$ GeV	D0 (3.6 fb <sup>-1</sup> ) $m_t = 174.94$ GeV
<b>DETECTOR RESPONSE</b>		
Jet energy scale		
Light-jet response (1)	0.41	n/a
Light-jet response (2)	0.01	0.63
Out-of-cone correction	0.27	n/a
Model for $b$ jets	0.23	0.07
Semileptonic $b$ decay	0.16	0.04
$b$ -jet hadronization	0.16	0.06
Response to $b/q/g$ jets	0.13	0.26
In-situ light-jet calibration	0.58	0.46
Jet modeling	0.00	0.36
Jet energy resolution	0.00	0.24
Jet identification	0.00	0.26
Lepton modeling	0.14	0.18
<b>MODELING SIGNAL</b>		
Signal modeling	0.56	0.77
Parton distribution functions	0.14	0.24
Quark annihilation fraction	0.03	n/a
Initial and final-state radiation	0.15	0.26
Higher-order QCD corrections	n/a	0.25
Jet hadronization and underlying event	0.25	0.58
Color reconnection	0.37	0.28
Multiple interactions model	0.10	0.05
<b>MODELING BACKGROUND</b>		
Background from theory	0.07	0.19
Higher-order correction for heavy flavor	0.03	0.07
Factorization scale for $W$ +jets	0.07	0.16
Normalization to predicted cross sections	0.25	0.07
Distribution for background	0.07	0.03
Background based on data	0.06	0.23
Normalization to data	0.00	0.06
Trigger modeling	0.00	0.06
$b$ -tagging modeling	0.00	0.10
Signal fraction for calibration	n/a	0.10
Impact of multijet background on the calibration	n/a	0.14
<b>METHOD OF MASS EXTRACTION</b>		
Calibration method	0.10	0.16
<b>STATISTICAL UNCERTAINTY</b>		
STATISTICAL UNCERTAINTY	0.65	0.83
<b>UNCERTAINTY ON JET ENERGY SCALE</b>		
UNCERTAINTY ON JET ENERGY SCALE	0.80	0.83
<b>OTHER SYSTEMATIC UNCERTAINTIES</b>		
OTHER SYSTEMATIC UNCERTAINTIES	0.67	0.94
<b>TOTAL UNCERTAINTY</b>		
TOTAL UNCERTAINTY	1.23	1.50

- Error budget in Tevatron determination  
CDF & D0 coll. 1207.1069
  - lepton+jets channel with matrix element method
- Modeling signal encompasses all perturbative uncertainties
  - radiative corrections (initial/final)
  - higher order QCD corrections
  - ...
- Uncertainties too optimistic  
 $\Delta m_t \simeq 150 \dots 250$  MeV
- Contradicts lattice bound  
 $\Delta m_t \geq 200$  MeV (if interpreted as pole mass)



# *Alternative methods*

- Top mass from leptonic decay:  $m_{lb}$  distribution
- Top mass from jet rates
- Top mass from total cross section



# Top mass from leptonic decay

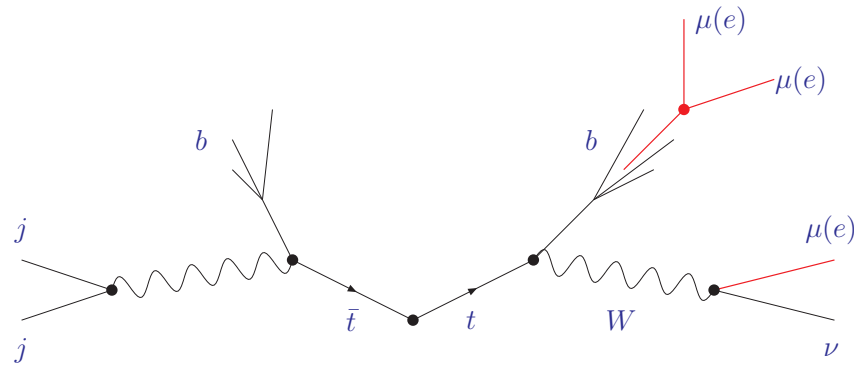
- Top mass from exclusive hadronic states

$$pp \rightarrow (t \rightarrow W^+ + b \rightarrow W^+ + J/\psi) + (\bar{t} \rightarrow W^- + \bar{b})$$

- identification of  $\mu$ -pair in  $J/\psi$  decay; leptonic or hadronic decay of  $W$

Kharchilava '00

Chierici, Dierlamm '06

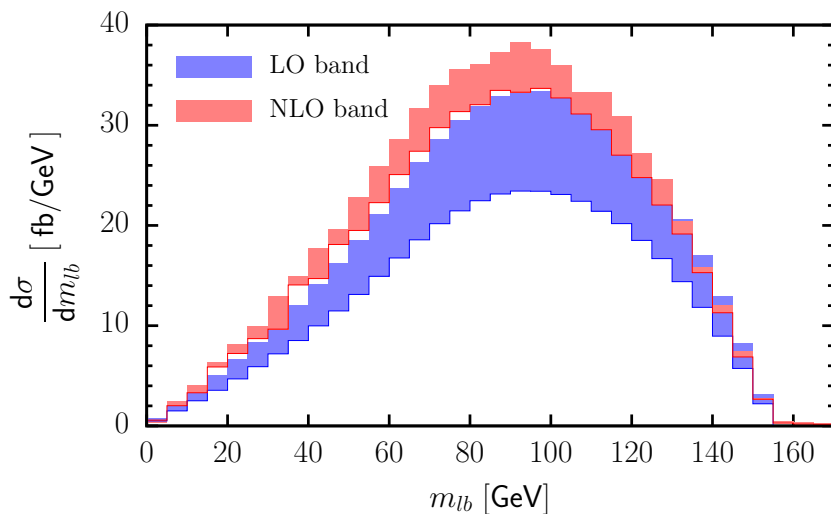


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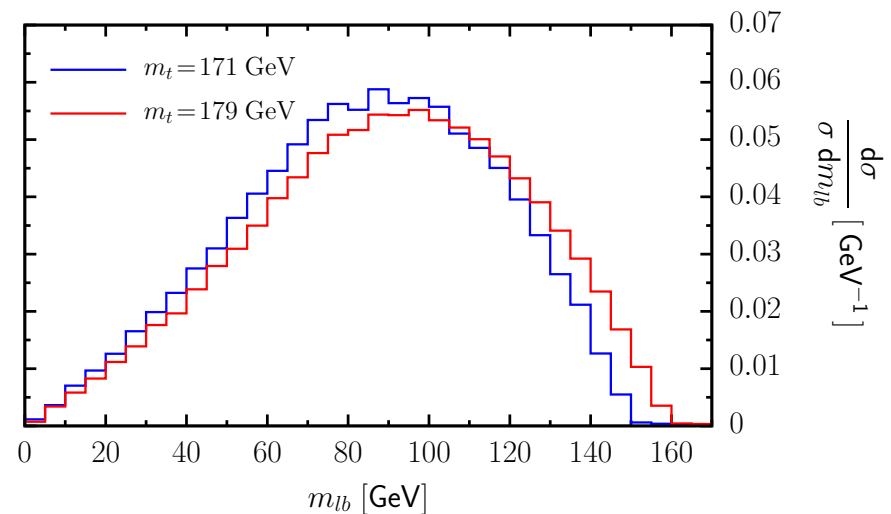
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$$pp \rightarrow (t \rightarrow W^+ + b \rightarrow W^+ + J/\psi) + (\bar{t} \rightarrow W^- + \bar{b})$$

- Study of  $m_{lb}$  distribution at NLO in QCD **Biswas, Melnikov, Schulze '10**
  - NLO QCD corrections to production and decay very important for value of  $m_t$  (effects of order  $\Delta m_t = \mathcal{O}(\text{few})$  GeV)
- Invariant mass distribution of lepton and  $b$ -jet (LHC14)
  - scale dependence at LO and NLO (left)
  - normalized  $m_{lb}$  distributions,  $m_t = 171$  GeV and 179 GeV (right)



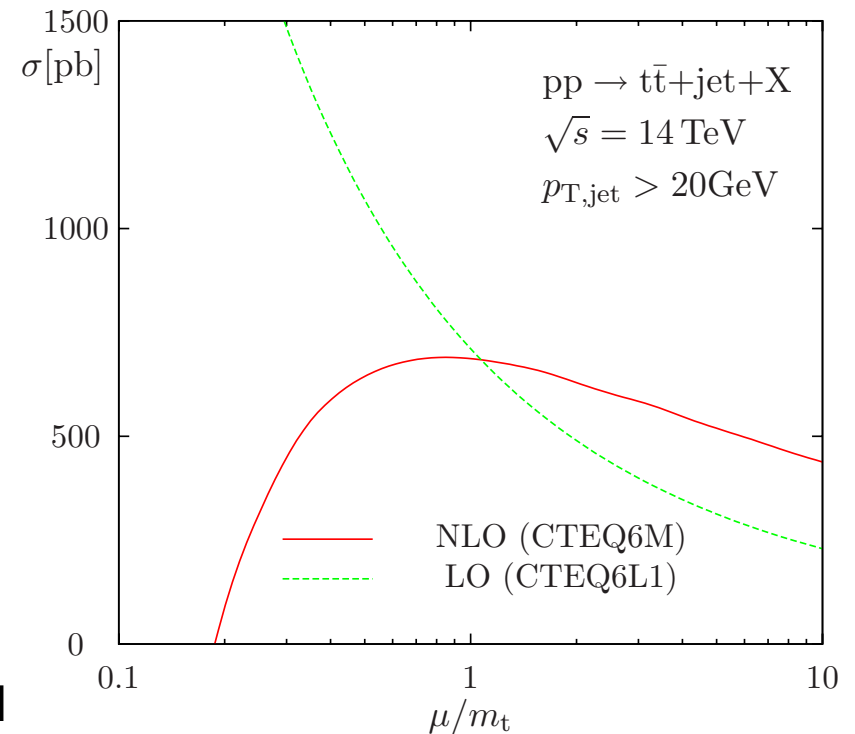
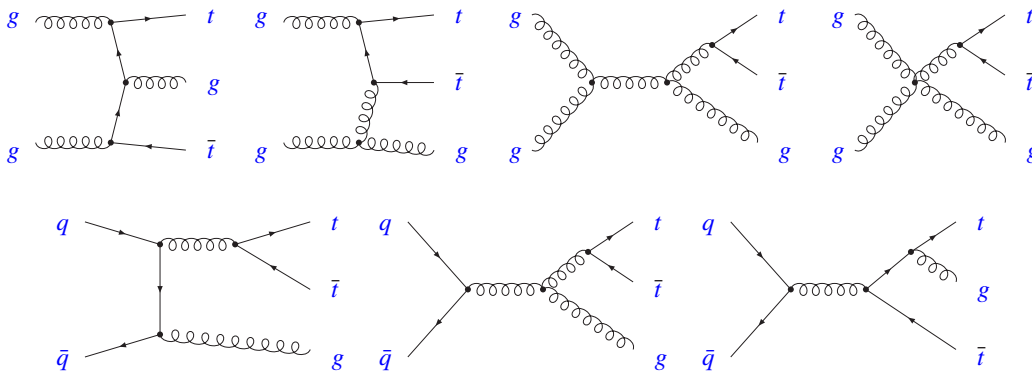
Sven-Olaf Moch



The Higgs-boson, the top-quark and electroweak vacuum stability – p.15

# Top-quark pairs with one jet

- LHC: large rates for production of  $t\bar{t}$ -pairs with additional jets
- NLO QCD corrections for  $t\bar{t} + 1\text{jet}$  *Dittmaier, Uwer, Weinzierl '07-'08*
  - scale dependence greatly reduced at NLO
  - corrections for total rate at scale  $\mu_r = \mu_f = m_t$  are almost zero



- Additional jet raises kinematical threshold
  - invariant mass  $\sqrt{s_{t\bar{t}+1\text{jet}}}$

# Mass measurement with $t\bar{t}$ + jet-samples

- Mass measurement with new observable

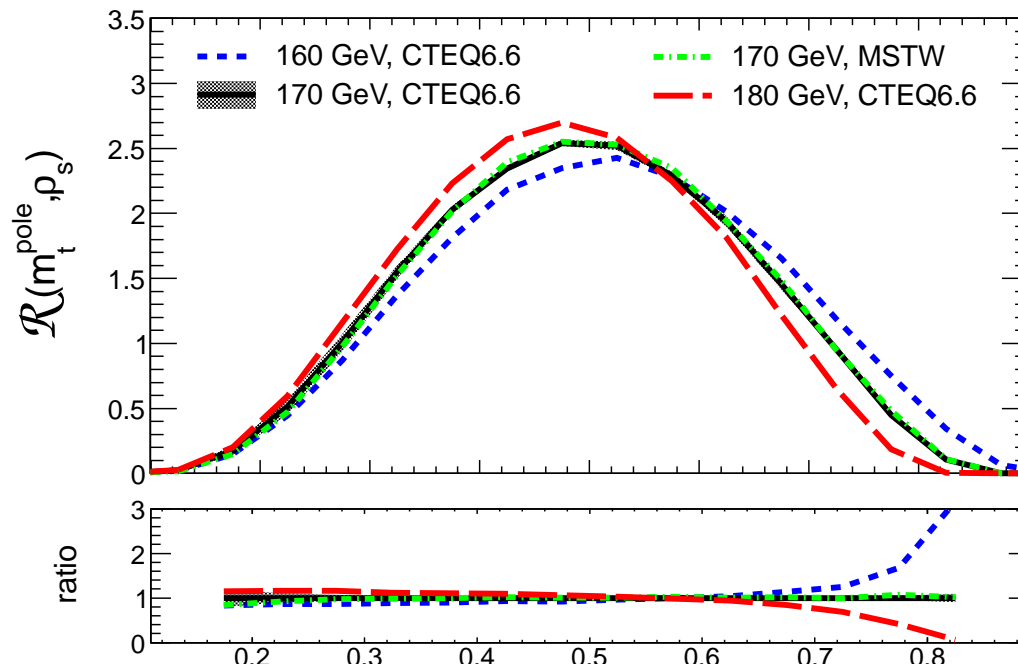
Alioli, Fernandez, Fuster, Irlles, S.M., Uwer, Vos '13

- variable  $\rho_s = \frac{2 \cdot m_0}{\sqrt{s_{t\bar{t}+1jet}}}$  with invariant mass of  $t\bar{t}$  + 1jet system and fixed scale  $m_0 = 170$  GeV

- Normalized-differential  $t\bar{t}$  + jet cross section

$$\mathcal{R}(m_t, \rho_s) = \frac{1}{\sigma_{t\bar{t}+1jet}} \frac{d\sigma_{t\bar{t}+1jet}}{d\rho_s}(m_t, \rho_s)$$

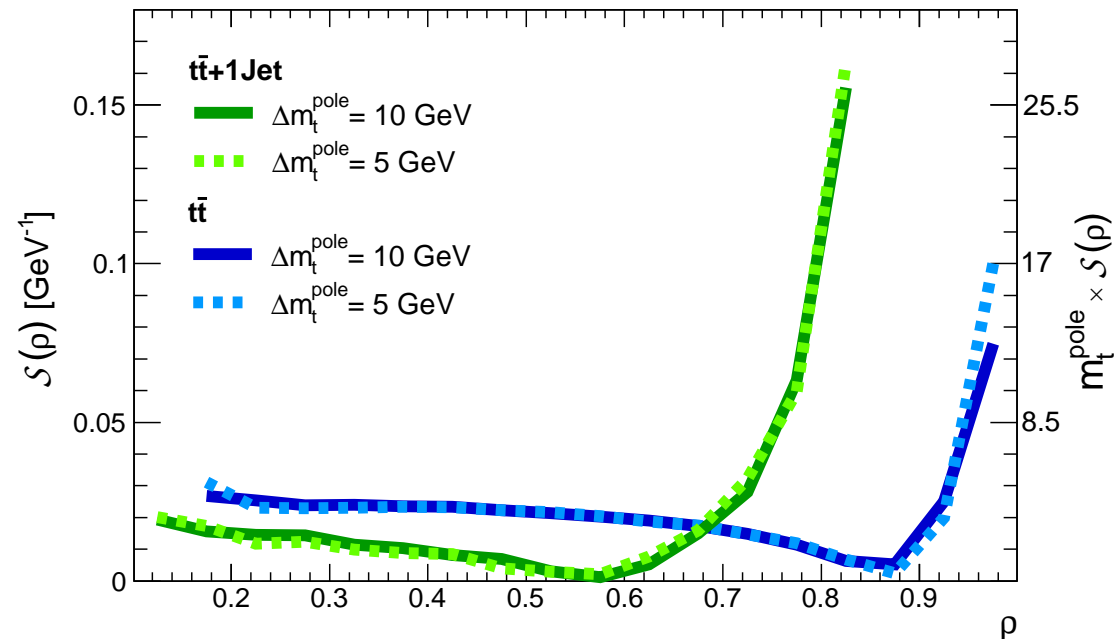
- significant mass dependence for  $0.4 \leq \rho_s \leq 0.5$  and  $0.7 \leq \rho_s$



- Differential cross section  $\mathcal{R}(m_t, \rho_s)$ 
  - good perturbative stability, small theory uncertainties, small dependence on experimental uncertainties, ...
- Sensitivity to top-quark mass very good

$$\left| \frac{\Delta \mathcal{R}}{\mathcal{R}} \right| \simeq (m_t \mathcal{S}) \times \left| \frac{\Delta m_t}{m_t} \right|$$

- increased sensitivity for system  $t\bar{t} + \text{jet}$  compared to  $t\bar{t}$

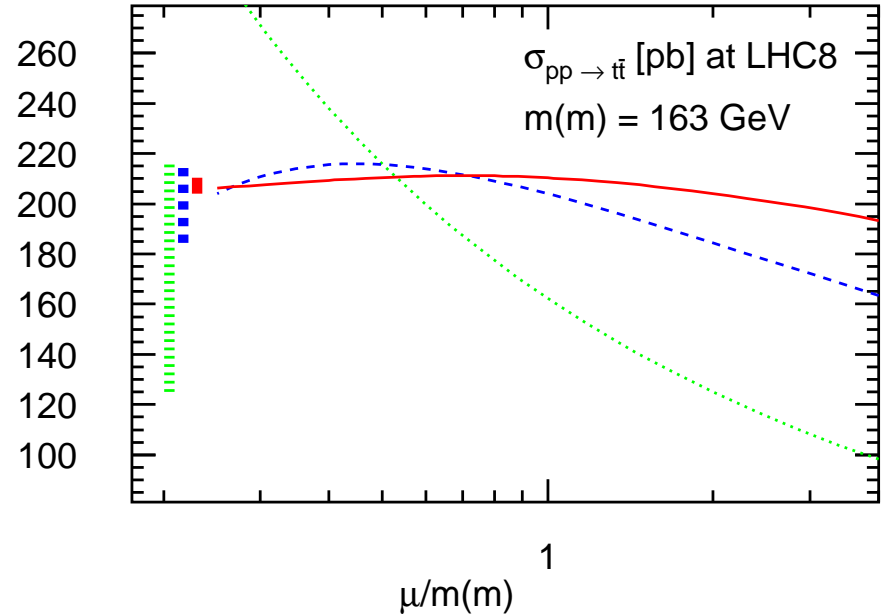
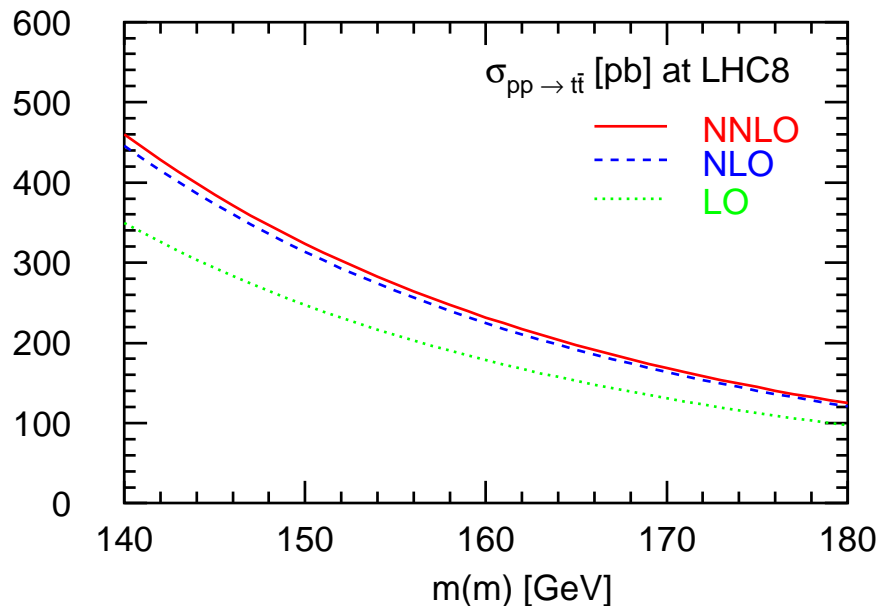


## Upshot

- Precision determination of well-defined top-quark mass  $m_t$  possible
  - alternative to inclusive cross sections

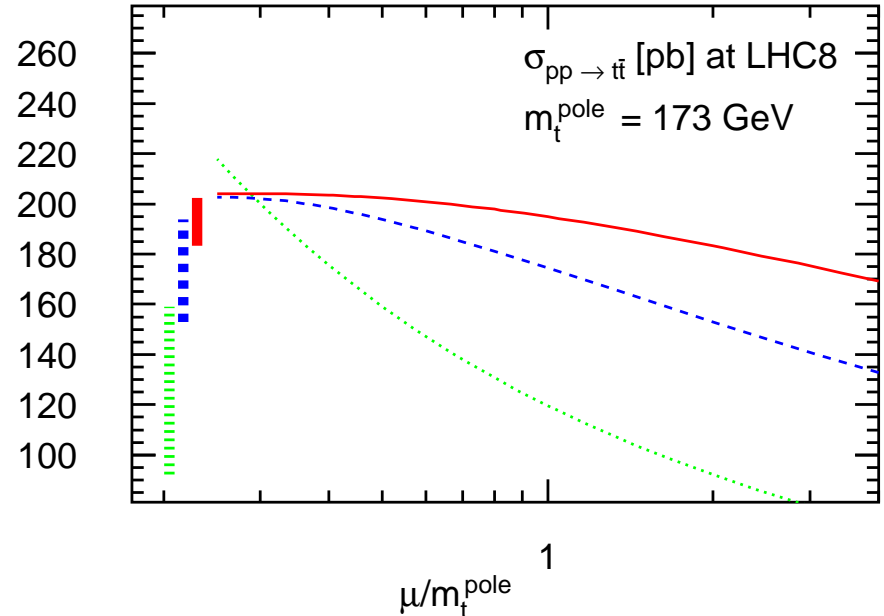
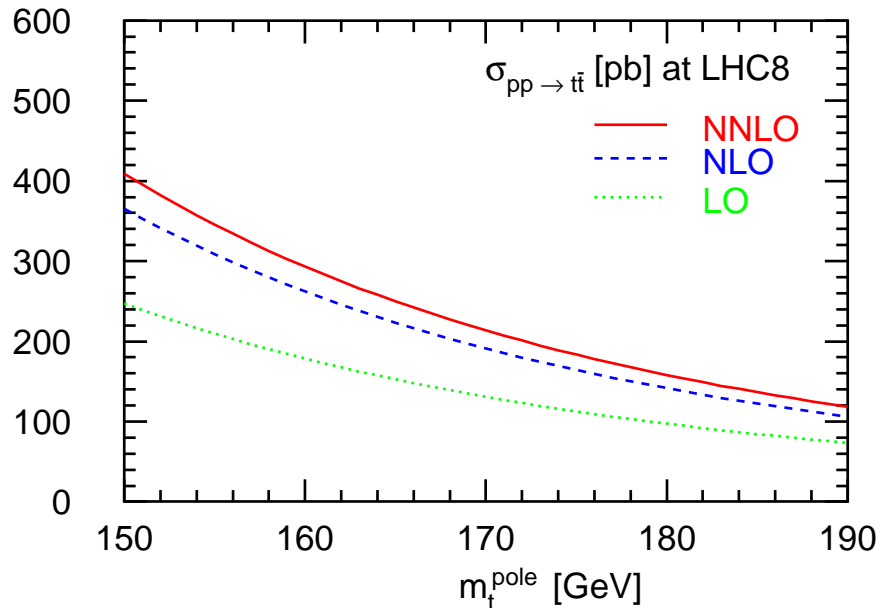
# Total cross section with $\overline{MS}$ mass

- $\overline{MS}$  mass definition  $m(\mu_R)$  realizes running mass (scale dependence)
  - short distance mass probes at scale of hard scattering
  - conversion between pole mass and  $\overline{MS}$  mass definition in perturbation theory:  $m_t = m(\mu_R) \left( 1 + a_s(\mu_R)d^{(1)} + a_s(\mu_R)^2 d^{(2)} \right)$
- Good apparent convergence of perturbative expansion
- Small theoretical uncertainty form scale variation

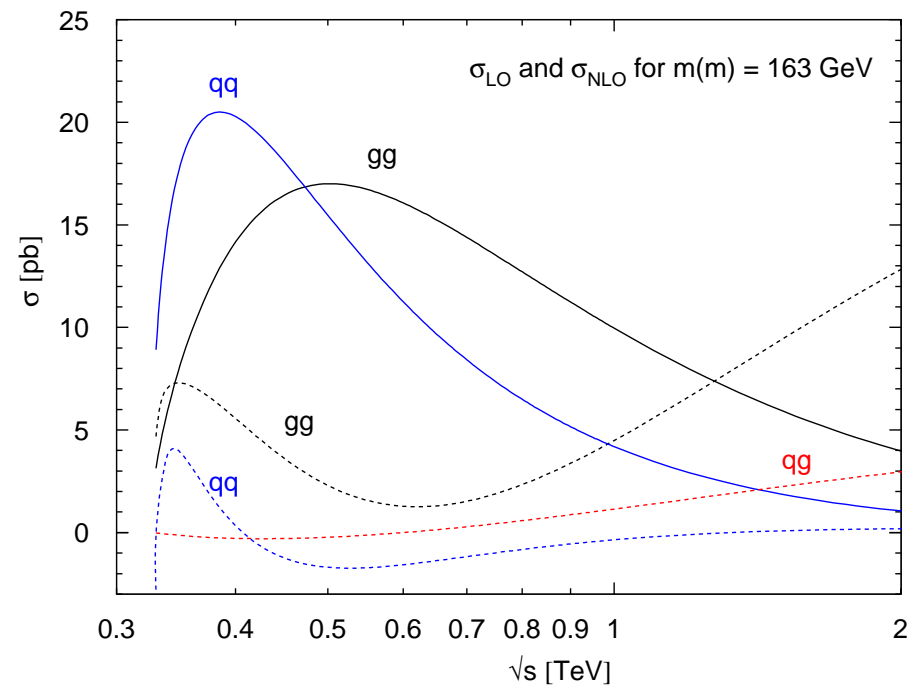
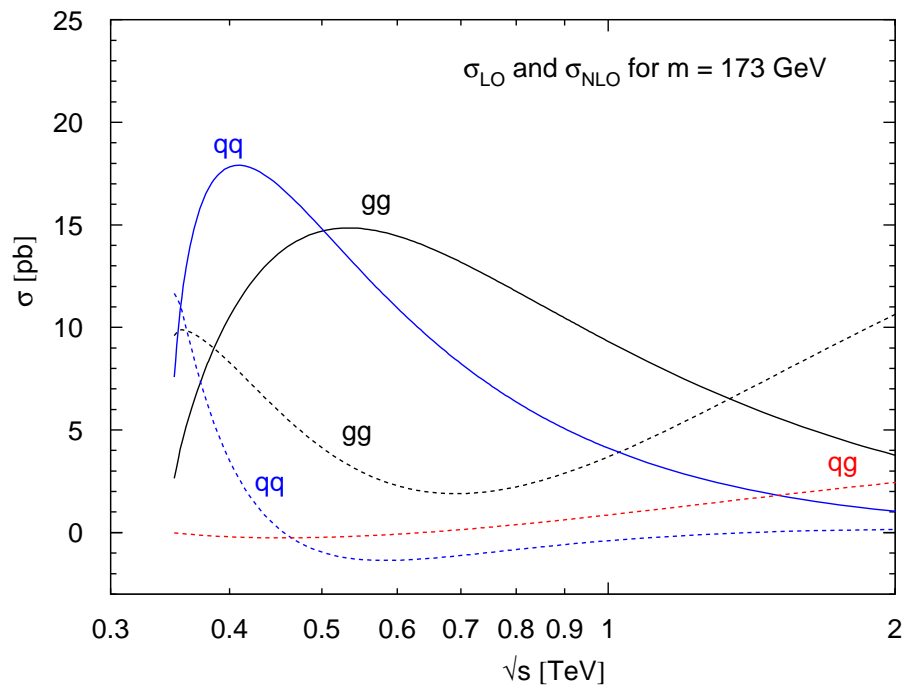


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- Pole mass scheme for comparison

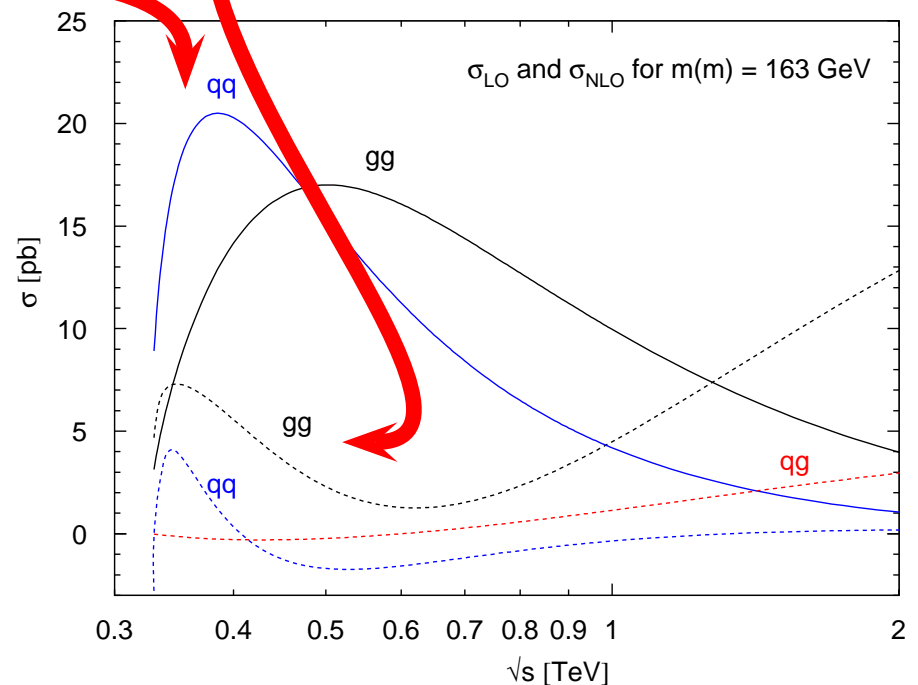
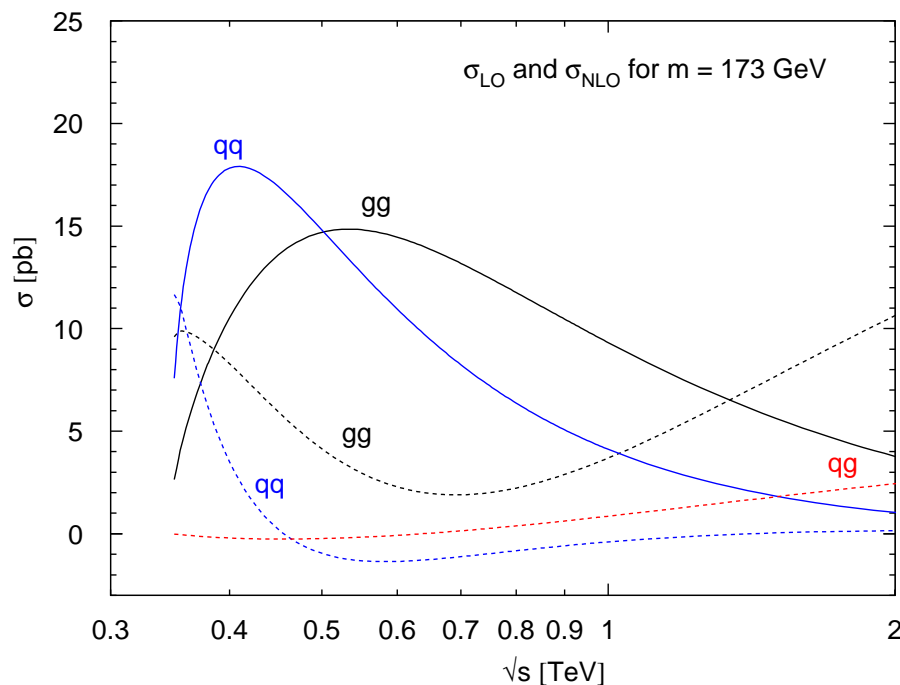


- Perturbative stability of predictions with  $\overline{MS}$  mass definition
- Parton cross section for channels  $q\bar{q}$ ,  $gg$  and  $qg$ 
  - on-shell scheme for  $m_t = 173$  GeV (left)
  - $\overline{MS}$  scheme for  $m(m) = 163$  GeV (right)





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  - $\overline{MS}$  scheme for  $m(m) = 163$  GeV (right)
- $\overline{MS}$  scheme
  - more emphasis on LO contribution
  - less significance to threshold region at NLO

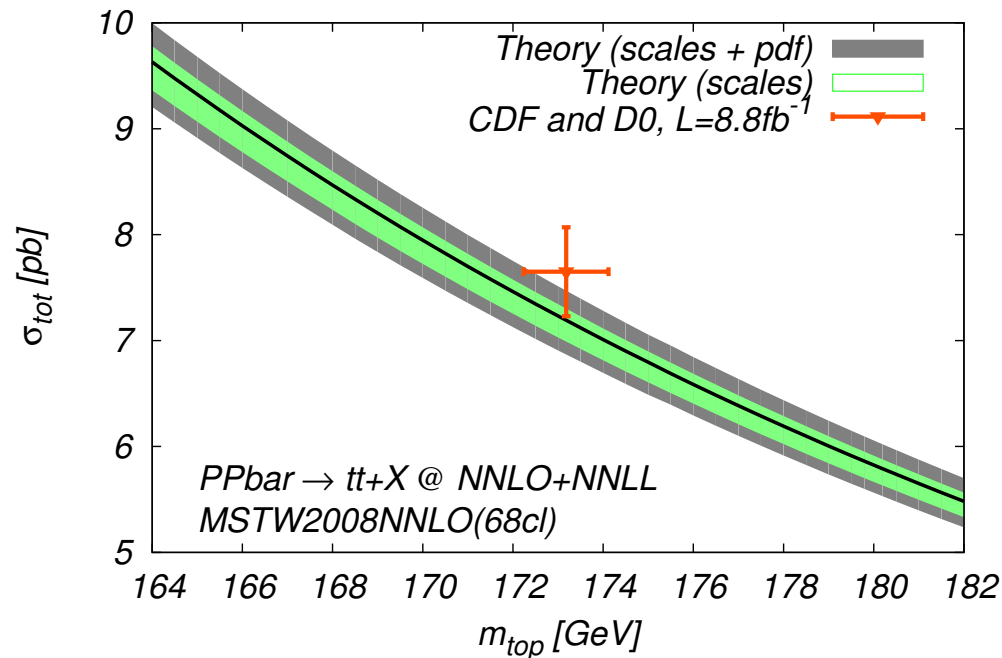


# Top mass from total cross section

## Exact result at NNLO in QCD

Czakon, Fiedler, Mitov '13

- Illustration of mass dependence for Tevatron (pole mass)



- NNLO perturbative corrections (e.g. at LHC8)
  - $K$ -factor (NLO  $\rightarrow$  NNLO) of  $\mathcal{O}(10\%)$
  - scale stability at NNLO of  $\mathcal{O}(\pm 5\%)$

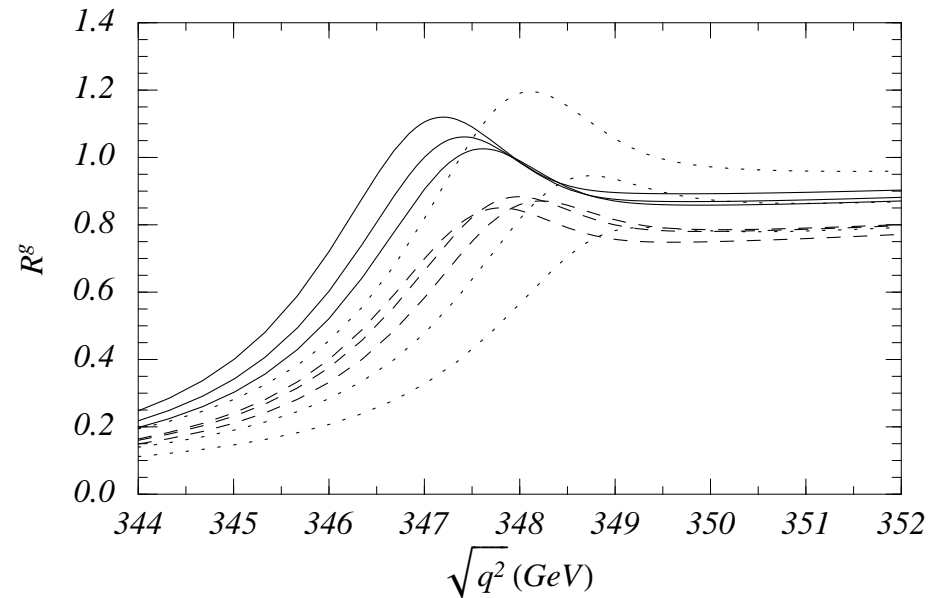
# Top quark pole mass

## Illustration for top quark pole mass

### ILC

- Pole mass measurements are strongly order-dependent

- e.g. threshold scan of cross section in  $e^+e^-$  collision
  - Beneke, Signer, Smirnov '99;
  - Hoang, Teubner '99;
  - Melnikov, Yelkhovsky '98;
  - Penin, Pivovarov '99;
  - Yakovlev '99
- LO (dotted), NLO (dashed), NNLO (solid)

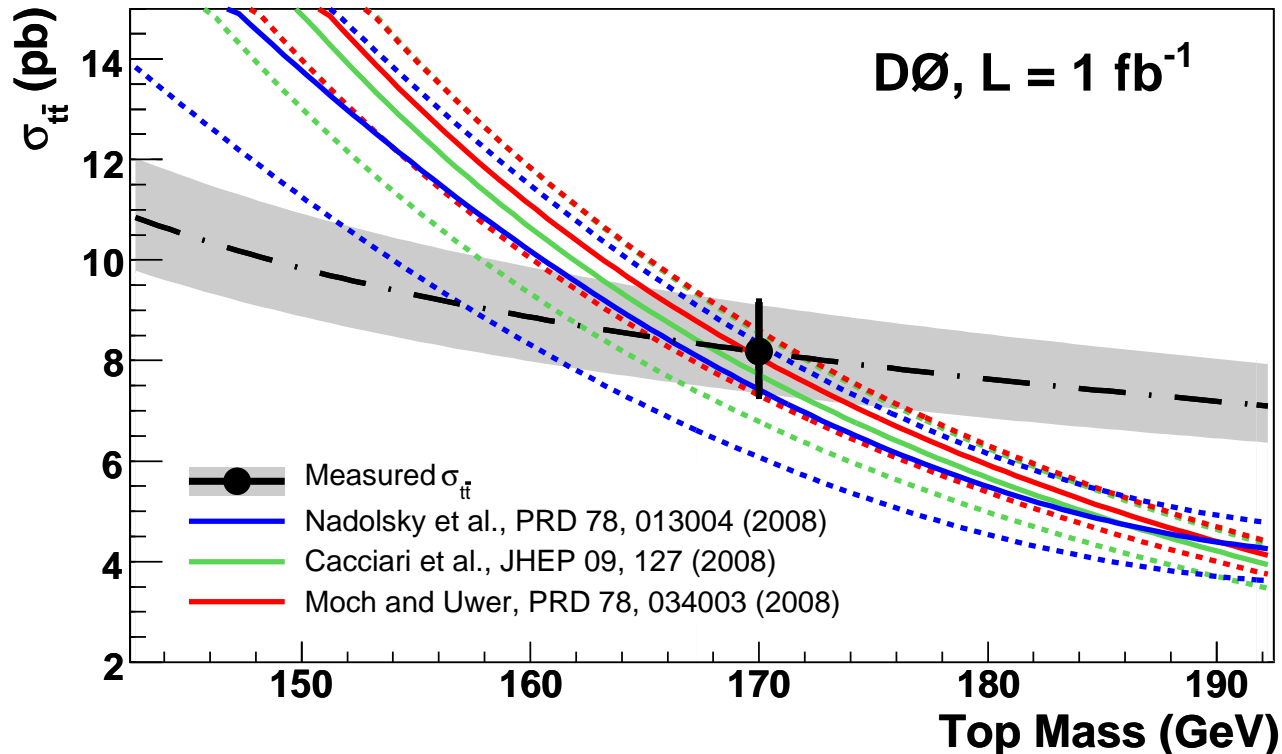


# Top quark pole mass

## Illustration for top quark pole mass

### Tevatron

- Total cross section and different channels of Tevatron analyses (theory uncertainty band from scale variation)
- Determination of  $m_t$  from total cross section (slope  $d\sigma/dm_t$ )
  - e.g. DZero '09: NLO  $m_t = 165.5^{+6.1}_{-5.9}$ ; NNLO  $m_t = 169.1^{+5.9}_{-5.2}$ ; ...



# Tevatron

- Determine top quark mass from Tevatron cross section data
  - $\sigma_{t\bar{t}} = 7.56^{+0.63}_{-0.56}$  pb D0 coll. arXiv:1105.5384
  - $\sigma_{t\bar{t}} = 7.50^{+0.48}_{-0.48}$  pb CDF coll. CDF-note-9913
- Fit of running mass  $m_t(m_t)$  for individual PDFs
  - parton luminosity at Tevatron driven by  $q\bar{q}$
  - $\overline{MS}$ -scheme for  $m_t^{\overline{MS}}(m_t)$ , then scheme transformation to pole mass  $m_t^{\text{pole}}$  at NNLO

	ABM11	JR09	MSTW08	NN21
$m_t^{\overline{MS}}(m_t)$	$162.0^{+2.3}_{-2.3}^{+0.7}_{-0.6}$	$163.5^{+2.2}_{-2.2}^{+0.6}_{-0.2}$	$163.2^{+2.2}_{-2.2}^{+0.7}_{-0.8}$	$164.4^{+2.2}_{-2.2}^{+0.8}_{-0.2}$
$m_t^{\text{pole}}$	$171.7^{+2.4}_{-2.4}^{+0.7}_{-0.6}$	$173.3^{+2.3}_{-2.3}^{+0.7}_{-0.2}$	$173.4^{+2.3}_{-2.3}^{+0.8}_{-0.8}$	$174.9^{+2.3}_{-2.3}^{+0.8}_{-0.3}$
$(m_t^{\text{pole}})$	$(169.9^{+2.4}_{-2.4}^{+1.2}_{-1.6})$	$(171.4^{+2.3}_{-2.3}^{+1.2}_{-1.1})$	$(171.3^{+2.3}_{-2.3}^{+1.4}_{-1.8})$	$(172.7^{+2.3}_{-2.3}^{+1.4}_{-1.2})$

- Good consistency within errors for  $m_t^{\text{pole}} = 171.7 \dots 174.9$  at NNLO

# The fine print

- Intrinsic limitation of sensitivity in total cross section

$$\left| \frac{\Delta\sigma_{t\bar{t}}}{\sigma_{t\bar{t}}} \right| \simeq 5 \times \left| \frac{\Delta m_t}{m_t} \right|$$

- Cross section at LHC has correlation of  $m_t$ ,  $\alpha_S(M_Z)$ , gluon PDF

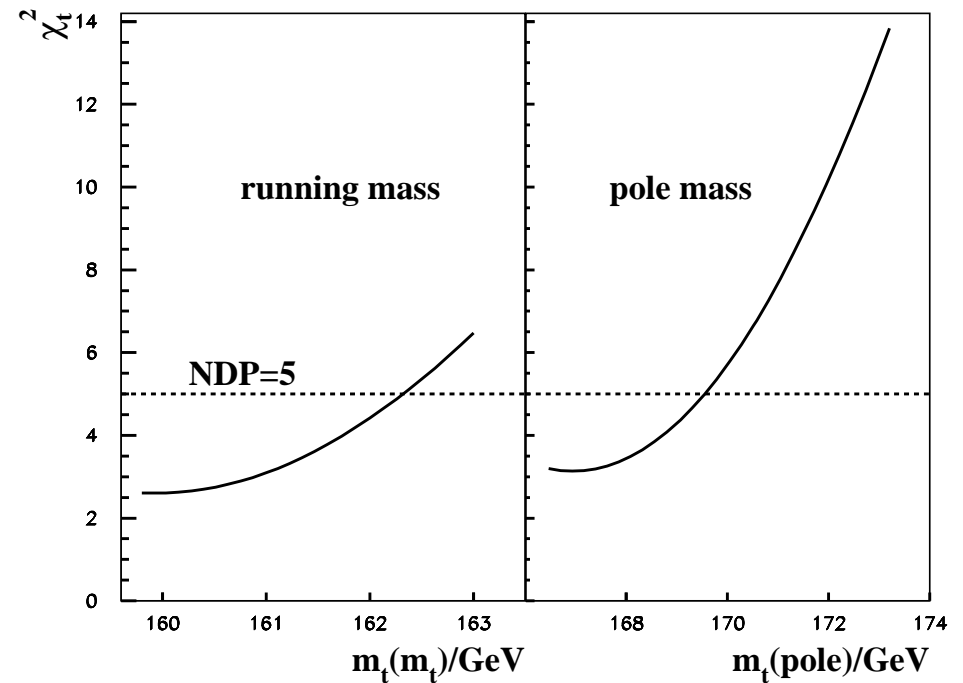
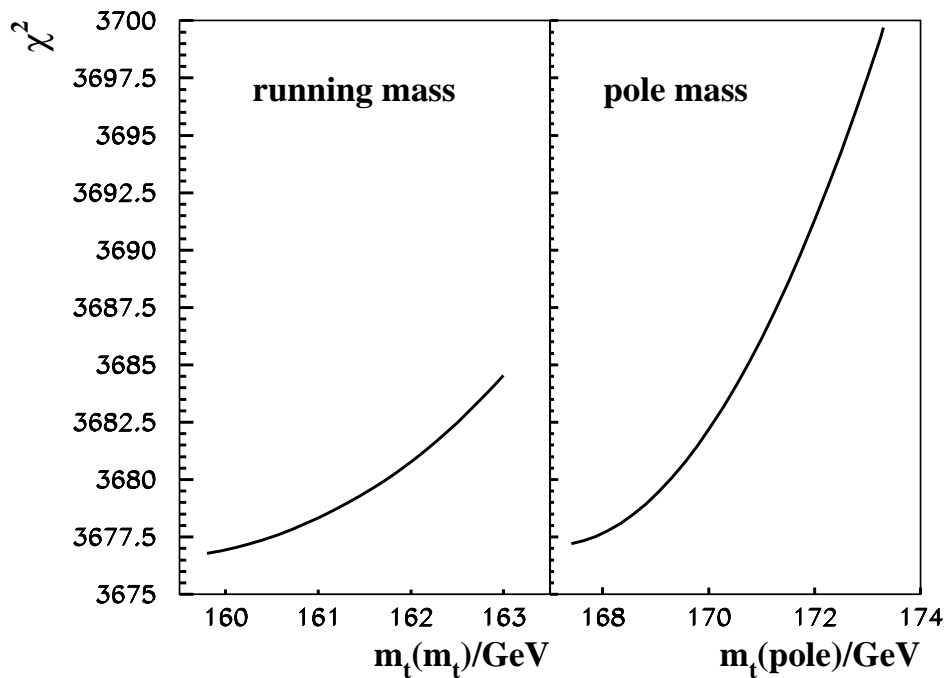
$$\sigma_{t\bar{t}} \sim \alpha_s^2 m_t^2 g(x) \otimes g(x)$$

- effective parton  $\langle x \rangle \sim 2m_t/\sqrt{s} \sim 2.5 \dots 5 \cdot 10^{-2}$
- fit with fixed values of  $m_t$  and  $\alpha_S(M_Z)$  carries significant bias

Czakon, Mangano, Mitov, Rojo '13

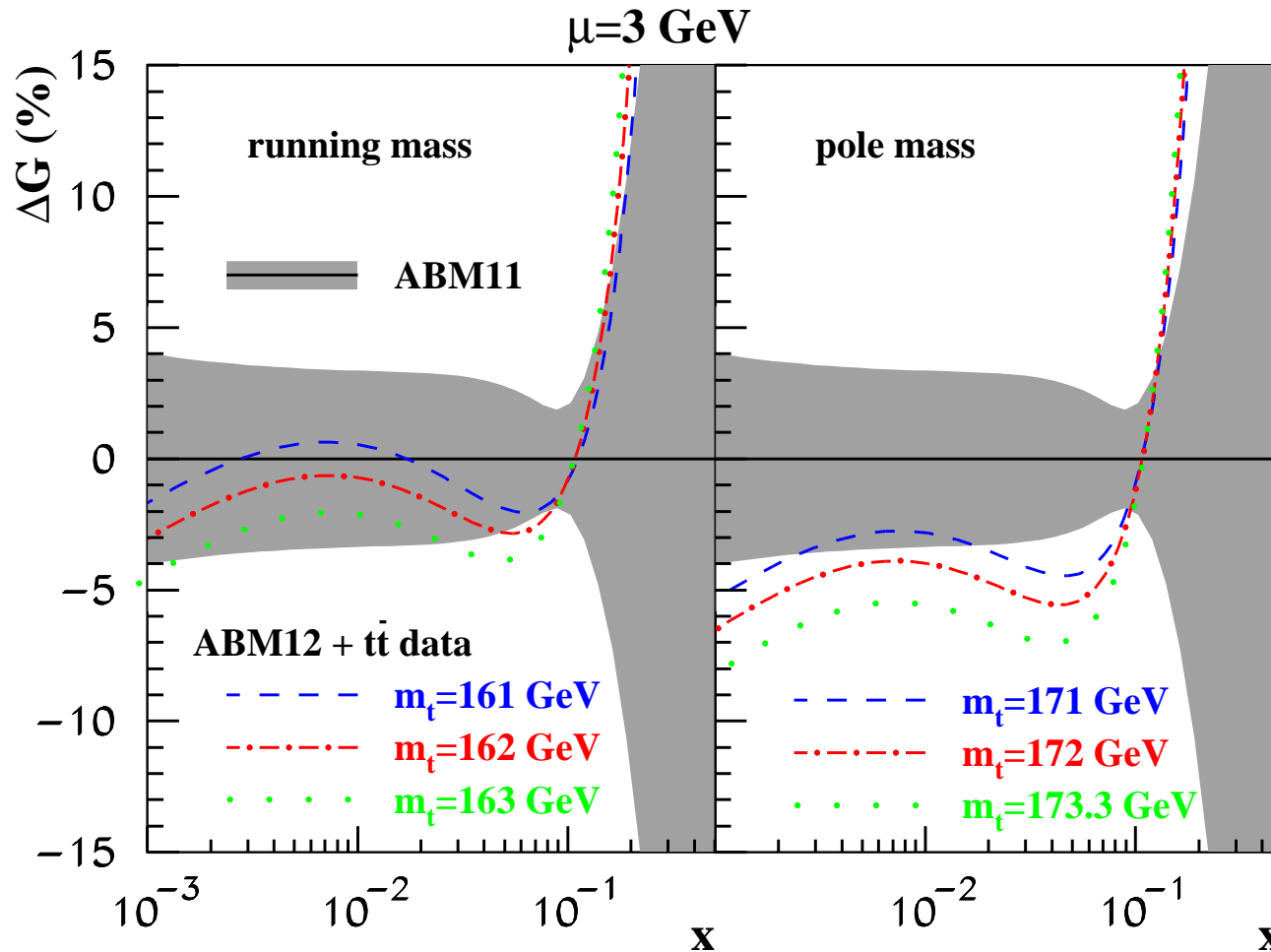
# The fine print

- Fit with correlations
  - $g(x)$  and  $\alpha_s(M_Z)$  already well constrained by global fit (no changes)
  - for fit with  $\chi^2/NDP = 5/5$  obtain value of  $m_t(m_t) = 162 \text{ GeV}$  Alekhin, Blümlein, S.M. '13
  - $\chi^2$ -profile steeper for pole mass (bigger impact of top-quark data)



# The fine print

- Fit with correlations
  - $g(x)$  and  $\alpha_s(M_Z)$  already well constrained by global fit (no changes)
  - correlation of gluon PDF with value of  $m_t$   
(illustration of bias in recent analysis [Czakon, Mangano, Mitov, Rojo '13](#))





# Higgs potential

## Renormalization group equation

- Quantum corrections to Higgs potential  $V(\Phi) = \lambda \left| \Phi^\dagger \Phi - \frac{v}{2} \right|^2$
- Radiative corrections to Higgs self-coupling  $\lambda$ 
  - electro-weak couplings  $g$  and  $g'$  of  $SU(2)$  and  $U(1)$
  - top-Yukawa coupling  $y_t$

$$16\pi^2 \frac{d\lambda}{dQ} = 24\lambda^2 - (3g'^2 + 9g^2 - 12y_t^2) \lambda + \frac{3}{8}g'^4 + \frac{3}{4}g'^2 g^2 + \frac{9}{8}g^4 - 6y_t^4 + \dots$$

# Higgs potential

## Triviality

- Large mass implies large  $\lambda$ 
  - renormalization group equation dominated by first term

$$16\pi^2 \frac{d\lambda}{dQ} \simeq 24\lambda^2 \quad \longrightarrow \quad \lambda(Q) = \frac{m_H^2}{2v^2 - \frac{3}{2\pi^2} m_H^2 \ln(Q/v)}$$

- $\lambda(Q)$  increases with  $Q$
- Landau pole implies cut-off  $\Lambda$ 
  - scale of new physics smaller than  $\Lambda$  to restore stability
  - upper bound on  $m_H$  for fixed  $\Lambda$

$$\Lambda \leq v \exp\left(\frac{4\pi^2 v^2}{3m_H^2}\right)$$

- Triviality for  $\Lambda \rightarrow \infty$ 
  - vanishing self-coupling  $\lambda \rightarrow 0$  (no interaction)

# Higgs potential

## Vacuum stability

- Small mass
  - renormalization group equation dominated by  $y_t$

$$16\pi^2 \frac{d\lambda}{dQ} \simeq -6y_t^4 \quad \longrightarrow \quad \lambda(Q) = \lambda_0 - \frac{\frac{3}{8\pi^2} y_0^4 \ln(Q/Q_0)}{1 - \frac{9}{16\pi^2} y_0^2 \ln(Q/Q_0)}$$

- $\lambda(Q)$  decreases with  $Q$
- Higgs potential unbounded from below for  $\lambda < 0$
- $\lambda = 0$  for  $\lambda_0 \simeq \frac{3}{8\pi^2} y_0^4 \ln(Q/Q_0)$
- Vacuum stability

$$\Lambda \leq v \exp\left(\frac{4\pi^2 m_H^2}{3y_t^4 v^2}\right)$$

- scale of new physics smaller than  $\Lambda$  to ensure vacuum stability
- lower bound on  $m_H$  for fixed  $\Lambda$

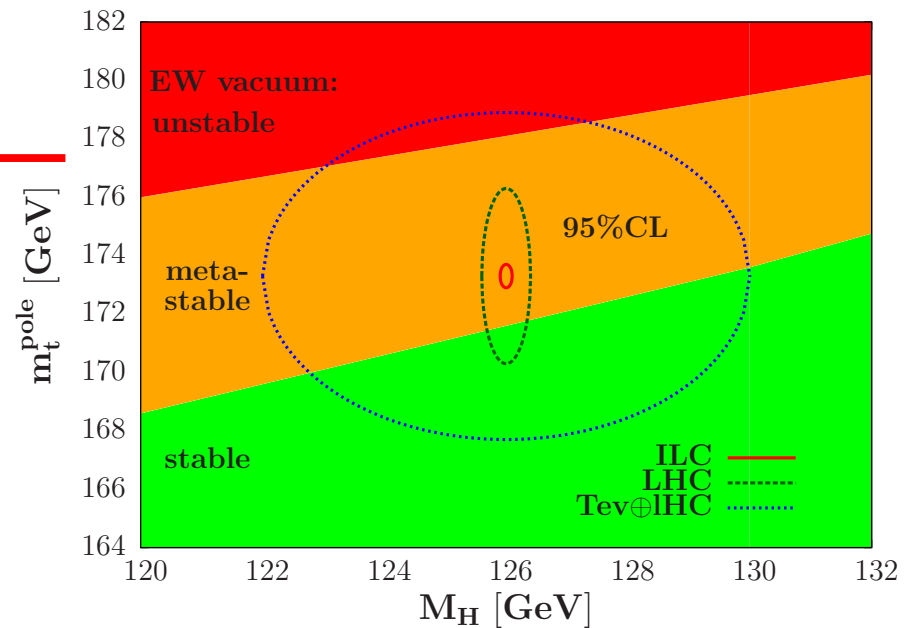
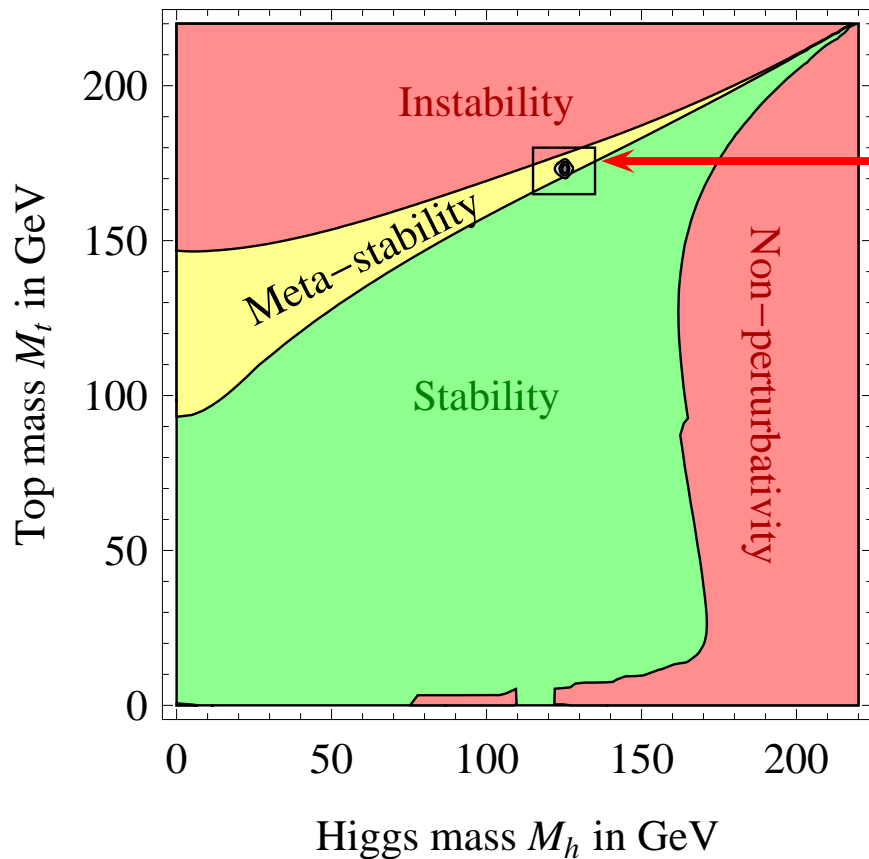
# Implications on electroweak vacuum

- Relation between Higgs mass  $m_H$  and top quark mass  $m_t$ 
  - condition of absolute stability of electroweak vacuum  $\lambda(\mu) \geq 0$
  - extrapolation of Standard Model up to Planck scale  $M_P$
  - $\lambda(M_P) \geq 0$  implies lower bound on Higgs mass  $m_H$

$$m_H \geq 129.2 + 1.8 \times \left( \frac{m_t^{\text{pole}} - 173.2 \text{ GeV}}{0.9 \text{ GeV}} \right) - 0.5 \times \left( \frac{\alpha_s(M_Z) - 0.1184}{0.0007} \right) \pm 1.0 \text{ GeV}$$

- recent NNLO analyses Bezrukov, Kalmykov, Kniehl, Shaposhnikov '12; Degrassi, Di Vita, Elias-Miro, Espinosa, Giudice et al. '12
  - uncertainty in results due to  $\alpha_s$  and  $m_t$  (pole mass scheme)
- Top quark mass from Tevatron in well-defined scheme
  - $m_t^{\overline{\text{MS}}}(m_t) = 163.3 \pm 2.7 \text{ GeV}$  implies in pole mass scheme  $m_t^{\text{pole}} = 173.3 \pm 2.8 \text{ GeV}$
  - good consistency of mass value between different PDF sets

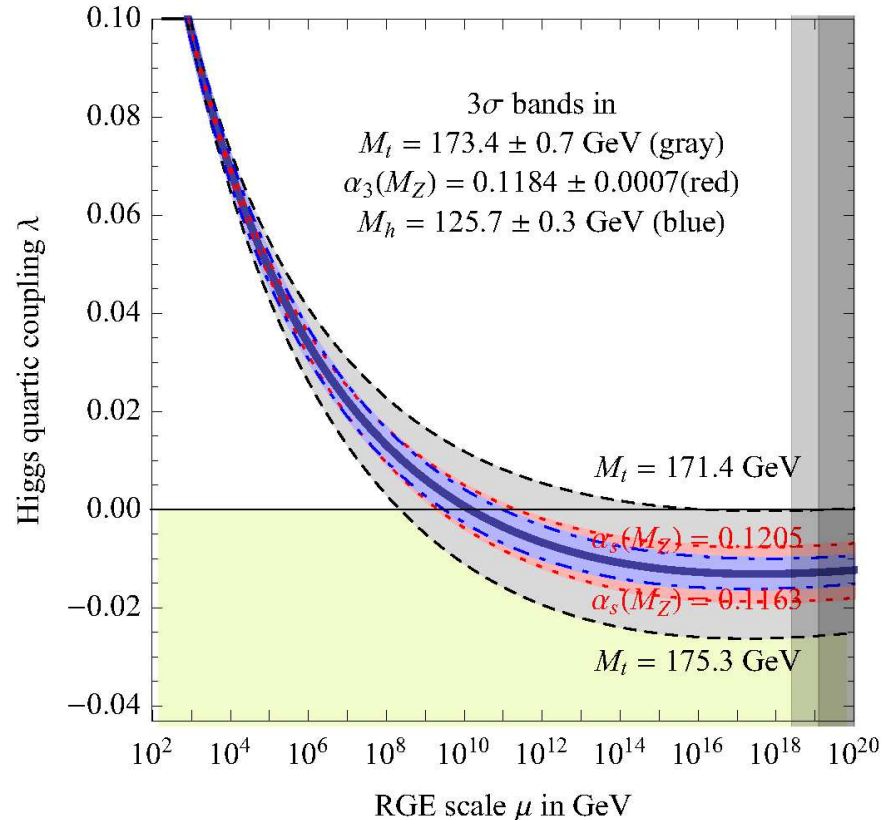
# Fate of the universe



Degrassi, Di Vita, Elias-Miro, Espinosa, Giudice et al. '12; Alekhin, Djouadi, S.M. '12; Masina '12

- Uncertainty in Higgs bound due to  $m_t$  from in  $\overline{MS}$  scheme
  - bound relaxes  $m_H \geq 129.4 \pm 5.6$  GeV
  - “fate of universe” still undecided

# Higgs self-coupling

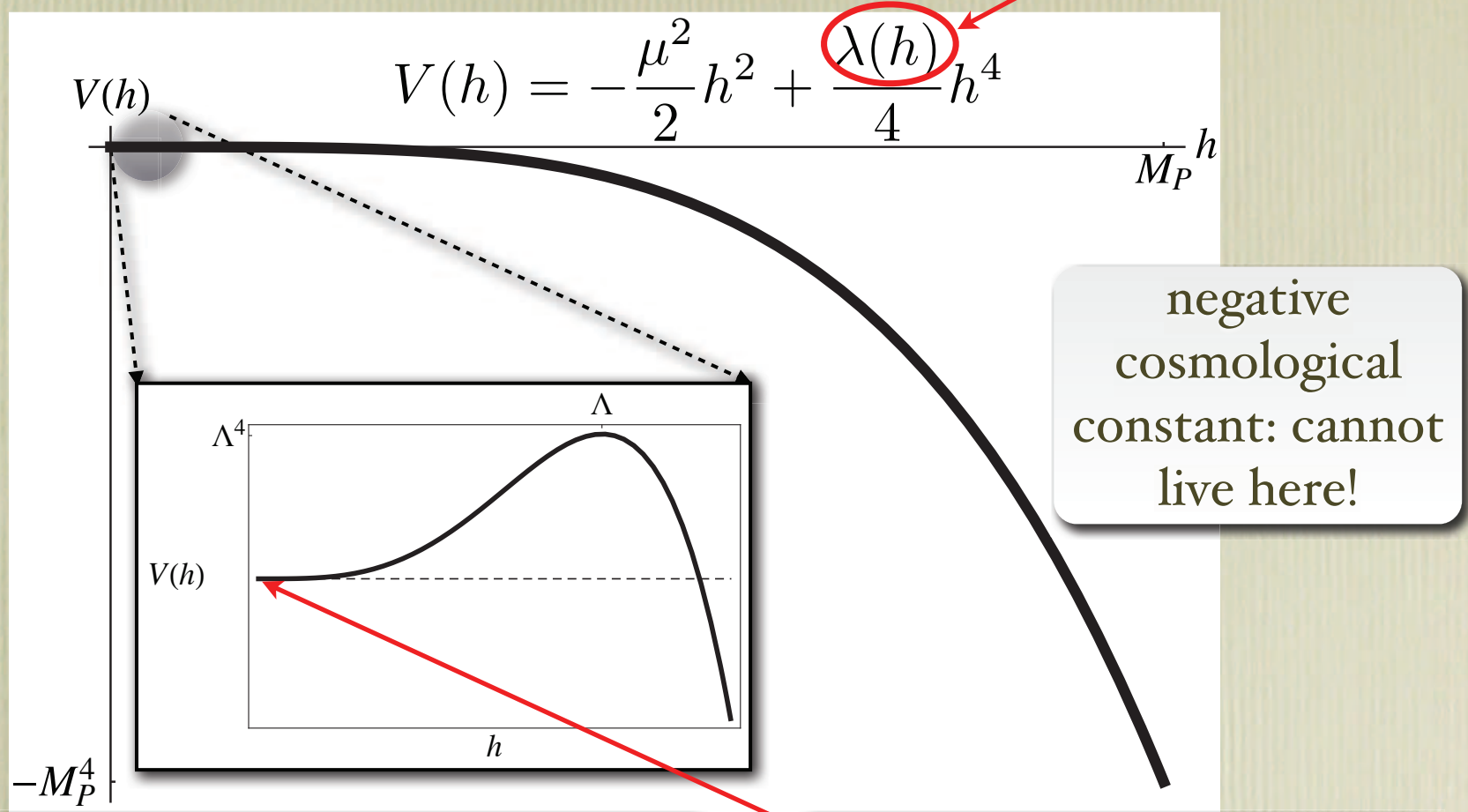


Buttazzo, Degrassi, Giardino, Giudice, Sala, Salvio, Strumia '13

- Renormalization group evolution of  $\lambda$  with uncertainties in  $m_H$ ,  $m_t$  and  $\alpha_s$ 
  - top-quark mass least precise parameter
- Vacuum stability bound at  $M_P$  in terms of  $m_t$

$$m_t \leq (171.36 \pm 0.15 \pm 0.25_{\alpha_3} \pm 0.17_{m_h}) \text{ GeV} = (171.36 \pm 0.46) \text{ GeV}$$

the Higgs scalar potential ... if the **coupling runs** negative!

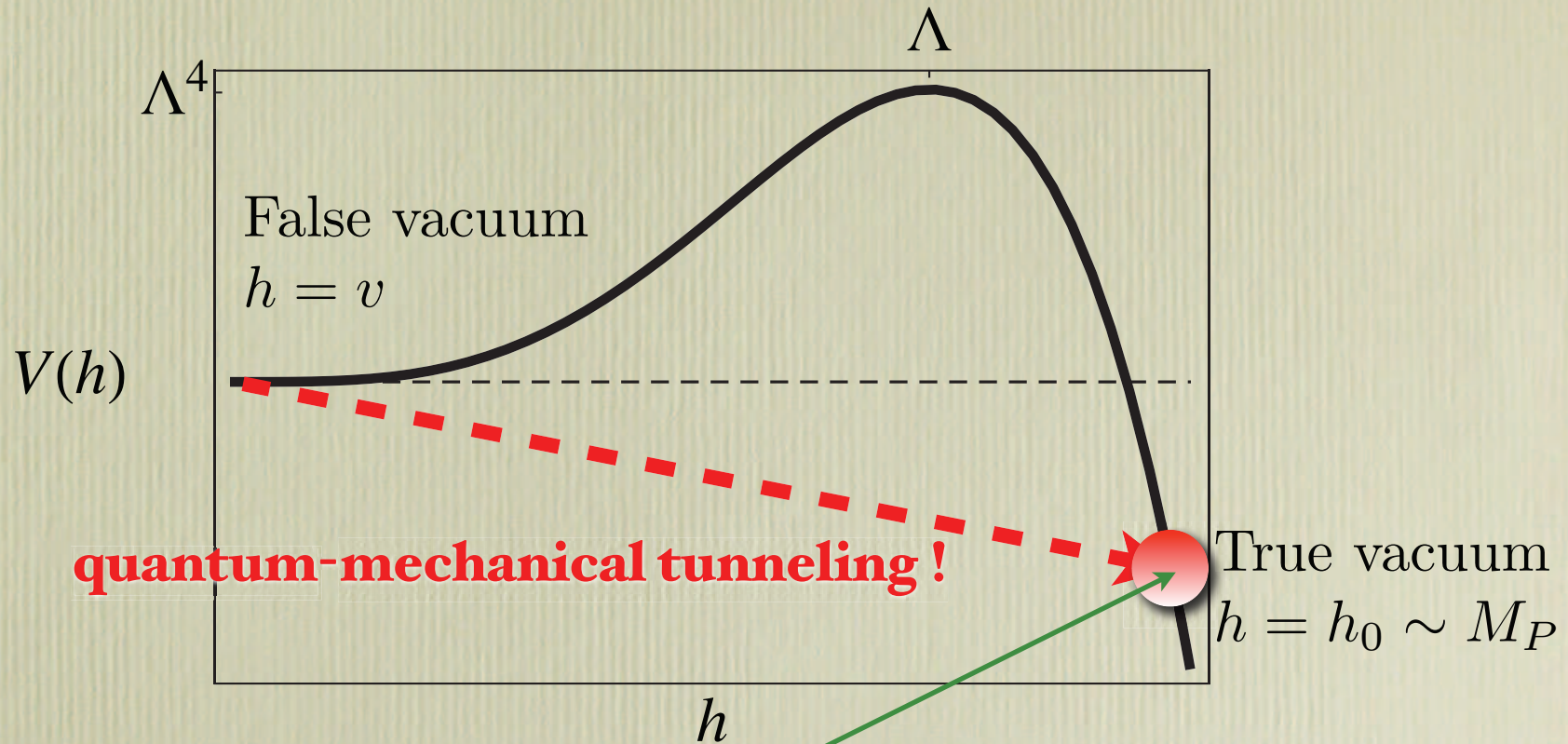


$\Lambda \sim 10^{10} \text{ GeV} \sim 10^{-8} M_P$   
 $\Lambda^4 \sim (10^{10} \text{ GeV})^4 \sim 10^{-32} M_P^4$

SM Higgs vacuum  
 $v \sim 10^2 \text{ GeV} \sim 10^{-16} M_P$

A. Westphal (DESY seminar 06/13)

# vacuum instability ...



quantum-mechanical tunneling!

True vacuum  
 $h = h_0 \sim M_P$

a bubble of true vacuum forms!

[Coleman & de Luccia '77; Arnold '89]  
[Isidori, Rychkov, Strumia & Tetradis '07]  
[Arkani-Hamed, Dubovsky, Senatore & Villadoro '08]

decay rate:

$$\Gamma \sim e^{-S_E} \ll \frac{1}{\text{age of Universe}}$$

A. Westphal (DESY seminar 06/13)



# Summary

## Top quark mass

- On-shell scheme (pole mass) at NNLO in QCD

$$m_t = 173.20 \pm 0.87 \pm \mathcal{O}(\text{few}) \text{ GeV}$$

- Running mass ( $\overline{\text{MS}}$  scheme) at NNLO in QCD

$$m_t(m_t) = 163.3 \pm 2.7 \text{ GeV}$$

# Summary

## Top quark mass

- Top quark mass is parameter of Standard Model Lagrangian
- Measurements of  $m_t$  require careful definition of observable
- Radiative corrections at higher orders mandatory for scheme definition

## Current measurements

- Kinematic reconstruction
  - very precise value, but only leading order/leading logarithm
  - lacking renormalization scheme definition
- $\overline{\text{MS}}$  mass from total cross section
  - NNLO QCD determination available
  - uncertainty  $\mathcal{O}(3)$  GeV from Tevatron analyses
  - LHC analyses affected by uncertainty in parton distributions,  $\alpha_S(M_Z)$

## Future challenge

- Study of new observables which meet all requirements
- Joint effort theory and experiment