## Higgs boson decay to $\gamma Z$ and test of CP and CPT symmetries

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## Outlook

- Introduction and motivations
- Effective Lagrangian for $h \gamma \gamma$ and $h \gamma Z$ interactions in the standard model (SM)
- Amplitudes for $h \rightarrow \gamma \gamma$ and $h \rightarrow \gamma Z$ decays
- Polarization parameters, density matrix
- Polarization parameters in models beyond the SM, discussion
- Conclusion


## Introduction

On 4 July 2012 the ATLAS and CMS collaborations announced discovery of new particle with mass about $125-126 \mathrm{GeV}$, properties of which correspond to expected Higgs boson of the SM. The spin is zero or two.
The $C P$ properties are not yet established, though data are more consistent with hypothesis of pure scalar boson, than the pseudoscalar one. In the decay channel to the pair of photons (status on July 2013):

$$
\begin{array}{r}
\mu_{\gamma \gamma}^{\exp } \equiv \frac{\Gamma^{\exp }(h \rightarrow \gamma \gamma)}{\Gamma^{\mathrm{SM}}(h \rightarrow \gamma \gamma)}=1.60 \pm 0.30, \\
\text { for } m_{h}=125.2 \pm 0.26(\text { stat })_{-0.6}^{+0.5}(\text { syst }) \mathrm{GeV}, \quad \text { ATLAS }
\end{array}
$$

- 

$$
\mu_{\gamma \gamma}^{\exp } \equiv \frac{\Gamma^{\exp }(h \rightarrow \gamma \gamma)}{\Gamma^{\mathrm{SM}}(h \rightarrow \gamma \gamma)}=0.77 \pm 0.27
$$

$$
\text { for } m_{h}=125.7 \pm 0.3 \text { (stat) } \pm 0.3 \text { (syst) } \mathrm{GeV}, \quad \mathrm{CMS}
$$

Although in SM the Higgs boson has $J^{P C}=0^{++}$, there are many extensions of the SM with a more complicated Higgs sector, where some of the Higgs bosons may have no definite $C P$-parity.

## Introduction

This aspect of the Higgs study is closely related to the origin of $C P$-violation.

- In the SM the source of violation of $C P$-symmetry is presence of unremovable phase in the Cabibbo-Kobayashi-Maskawa matrix. All data indicate that the CKM phase is the dominant source of $C P$-violation in flavor changing processes.
- However, calculations show that this $C P$ violation in SM is far too small to explain matter-antimatter asymmetry in the Universe. Thus there can be additional sources of $C P$ violation.
- It can be that $C P$-violating effects in the Higgs sector can be discovered only in high-energy processes with creation of Higgs boson(s).
- It also seems possible that $C P$ violation in the Higgs sector will be found in low-energy processes, e.g. in decays of $K$ - or $B$-mesons, or neutron dipole moment.


## CPT symmetry

One of the deep results of Quantum Field Theory is the CPT theorem [G. Luders 1952, W. Pauli 1957]:
any theory is symmetric under combined discrete transformation $C P T$, if it satisfies:

- Lorentz invariance,
- locality,
- quantum mechanical evolution of a system (Hermiticity of Hamiltonian),
- integer (half-integer) spin particles are subject to Bose-Einstein (Fermi-Dirac) statistics.

The consequences of the CPT symmetry are:
(i) $m_{p}=m_{\bar{p}}$,
(ii) $\tau_{p}=\tau_{\bar{p}}$,
(iii) $\mu_{p}=-\mu_{\bar{p}}$.

The best experimental check is performed for neutral kaons $K^{0}, \bar{K}^{0}$ with fantastic precision:

$$
\frac{m_{K^{0}}-m_{\bar{K}^{0}}}{m_{K^{0}}}<3 \times 10^{-19}
$$

## The CPT symmetry may be violated (?)

However, CPT can reveal itself in other ways, thus stimulating studies of various extensions of the SM in which CPT violation appears due to:

- nonlocality in the string theory,
- violation of Lorentz symmetry in models with extra dimensions,
- possible deviations from standard quantum mechanical evolution of states (violation of hermiticity or unitarity) in some models of quantum gravity.
[see series of Meetings on CPT and Lorentz Symmetry 2008, 2009, 2010, 2011, 2012, 2013, organized by A. Kostelecky, http://www.physics.indiana.edu/ kostelec/faq.html]

We point out that the study of decay of the discovered boson to photon and $Z$-бозон ( $h \rightarrow \gamma Z$ ) can be useful to clarify the CP properties of this boson, and possibly to test validity of the CPT theorem.

As for experiment on $h \rightarrow \gamma Z$ : preliminary results of ATLAS and CMS indicate that experimental limits are about an order of magnitude larger than the SM expectation $\left(B R(h \rightarrow \gamma Z) \approx 1.5 \times 10^{-3}\right)$. This process will be studied in detail after the LHC upgrade.

## Effective Lagrangian

Effective Lagrangian for $h \gamma \gamma$ and $h \gamma Z$ interaction can be written as

$$
\begin{gathered}
\mathcal{L}_{\mathrm{eff}}^{h \gamma \gamma}=\frac{e^{2}}{32 \pi^{2} v}\left(c_{\gamma} F_{\mu \nu} F^{\mu \nu} h-\tilde{c}_{\gamma} F_{\mu \nu} \widetilde{F}^{\mu \nu} h\right) \\
\mathcal{L}_{\mathrm{eff}}^{h \gamma Z}=\frac{e g}{16 \pi^{2} v}\left(c_{1 Z} Z_{\mu \nu} F^{\mu \nu} h-c_{2 Z}\left(\partial_{\mu} h Z_{\nu}-\partial_{\nu} h Z_{\mu}\right) F^{\mu \nu}-\tilde{c}_{Z} Z_{\mu \nu} \widetilde{F}^{\mu \nu} h\right)
\end{gathered}
$$

where $e$ is electric charge of positron, $g-S U(2)_{L}$ coupling constant, $v=\left(\sqrt{2} G_{F}\right)^{-1 / 2} \approx 246 \mathrm{GeV}$ is vacuum expectation value of scalar field. Here the field strengths for the electromagnetic and $Z$ field, and dual tensor are

$$
F_{\mu \nu}=\partial_{\mu} A_{\nu}-\partial_{\nu} A_{\mu}, \quad Z_{\mu \nu}=\partial_{\mu} Z_{\nu}-\partial_{\nu} Z_{\mu} \quad \widetilde{F}_{\mu \nu}=\frac{1}{2} \varepsilon_{\mu \nu \alpha \beta} F^{\alpha \beta}
$$

The terms $c_{\gamma}, c_{1 z}$, and $c_{2 z}$ correspond to a $C P$-even scalar $h$, while $\tilde{c}_{\gamma}$ and $\tilde{c}_{z}$ indicate a $C P$-odd pseudoscalar $h$. The presence of both sets means that $h$ is not a $C P$ eigenstate. Interference of these terms lead to $C P$ violating effects which reveal in polarization states of the photon.

## Effective coupling constants

Dimensionless parameters $c_{\gamma}, c_{1 Z}, c_{2 Z}, \tilde{c}_{\gamma}$, and $\tilde{c}_{Z}$ are effective coupling constants. They are, in general, complex-valued, so that effective Lagrangian is non-Hermitian, while being local and Lorentz invariant.

In general these constants contain the SM and NP contributions,

$$
c_{\gamma}=c_{\gamma}^{\mathrm{SM}}+c_{\gamma}^{\mathrm{NP}}, \quad c_{1 Z}=c_{Z}^{\mathrm{SM}}+c_{1 Z}^{\mathrm{NP}} .
$$

In the $\mathrm{SM} \tilde{c}_{\gamma}=c_{2 Z}=\tilde{c}_{Z}=0$, and thus

$$
\tilde{c}_{\gamma}=\tilde{c}_{\gamma}^{N P}, \quad c_{2 Z}=c_{2 Z}^{N P}, \quad \tilde{c}_{Z}=\tilde{c}_{Z}^{N P} .
$$

## Lowest-order diagrams for $h \rightarrow \gamma+\gamma(Z)$ in the SM

From vertices of the SM one calculates $h \rightarrow \gamma+\gamma(Z)$ one-loop diagrams [L. Bergström, G. Hulth: 1985, 1986 (Errata), M. Spira: 1998]


In the SM there is only $C P$-even terms in effective Lagrangian. The reason is that the axial-vector part in the $Z f \bar{f}$ coupling

$$
-i \frac{g}{2 \cos \theta_{W}} \gamma^{\mu}\left(g_{V}-g_{A} \gamma^{5}\right)
$$

does not contribute due to Furry's theorem.

## Coefficients in the SM in one-loop order

$$
\begin{gathered}
c_{\gamma}^{\mathrm{SM}}=A_{1}^{\gamma}\left(\tau_{W}\right)+\sum_{f=(\ell, q)} N_{f} Q_{f}^{2} A_{1 / 2}^{\gamma}\left(\tau_{f}\right) \approx-6.60+0.08 i \\
c_{Z}^{\mathrm{SM}}=-A_{1}^{Z}\left(\tau_{W}, \lambda_{W}\right)-\sum_{f=(\ell, q)} N_{f} Q_{f} g_{f} A_{1 / 2}^{Z}\left(\tau_{f}, \lambda_{f}\right) \approx-5.540+0.005 i
\end{gathered}
$$

$N_{f}=1(3)$ for leptons (quarks), $Q_{f}$ is fermion charge in units $e$, $g_{f}=\left(2 t_{3 L, f}-4 Q_{f} \sin ^{2} \theta_{W}\right) / \cos \theta_{w}$, where $t_{3 L, f}$ is projection of weak isospin, $\theta_{w}$ is weak angle. One-loop functions: $A_{1}^{\gamma}, A_{1 / 2}^{\gamma}, A_{1}^{Z}, A_{1 / 2}^{Z}$ depend on values of masses through parameters:
$\tau_{W}=4 m_{W}^{2} / m_{h}^{2}, \quad \lambda_{W}=4 m_{W}^{2} / m_{Z}^{2}, \quad \tau_{f}=4 m_{f}^{2} / m_{h}^{2}, \quad \lambda_{f}=4 m_{f}^{2} / m_{Z}^{2}$,
There are small imaginary parts coming from intermediate on-mass-shell states of leptons $e, \mu, \tau$ and quarks (except top quark), i.e. processes $h \rightarrow f \bar{f} \rightarrow \gamma \gamma(Z)$ with $m_{f} \leq m_{h} / 2$.
Couplings $c_{\gamma}^{\mathrm{NP}}, c_{1 Z}^{\mathrm{NP}}, \quad \tilde{c}_{\gamma}, c_{2 Z}, \quad \tilde{c}_{Z}$, originating from physics beyond the SM, will be estimated below.

## Amplitudes and polarizations for $h \rightarrow \gamma \gamma$

For decay of the zero-spin $h$ boson into a pair of photons

$$
h(p) \rightarrow \gamma\left(k_{1}, \epsilon_{1}\right)+\gamma\left(k_{2}, \epsilon_{2}\right) .
$$

In the rest frame of $h$, the amplitude is

$$
\mathcal{A}(h \rightarrow 2 \gamma)=\frac{e^{2} m_{h}^{2}}{16 \pi^{2} v}\left(c_{\gamma}\left(\vec{e}_{1}^{*} \vec{e}_{2}^{*}\right)+\tilde{c}_{\gamma}\left(\hat{\vec{k}}\left[\vec{e}_{1}^{*} \times \vec{e}_{2}^{*}\right]\right)\right)
$$

Helicity amplitudes are equal to

$$
H_{ \pm}=-\frac{e^{2} m_{h}^{2}}{16 \pi^{2} v}\left(c_{\gamma} \pm i \tilde{c}_{\gamma}\right)
$$

with the decay width

$$
\Gamma(h \rightarrow 2 \gamma)=\frac{1}{32 \pi m_{h}}\left(\left|H_{+}\right|^{2}+\left|H_{-}\right|^{2}\right) \sim\left|c_{\gamma}\right|^{2}+\left|\tilde{c}_{\gamma}\right|^{2}
$$

## Photon polarization parameters

The polarization states of a single photon are usually described in terms of density matrix $\rho^{(\gamma)}$ with the Stokes parameters $\zeta_{1}, \zeta_{2}$ and $\zeta_{3}$ :

$$
\rho_{\lambda \lambda^{\prime}}^{(\gamma)}=\frac{1}{2}\left(\begin{array}{cc}
1+\zeta_{2} & -\zeta_{3}+i \zeta_{1} \\
-\zeta_{3}-i \zeta_{1} & 1-\zeta_{2}
\end{array}\right)=\frac{1}{2}\left(1-\zeta_{3} \sigma_{1}-\zeta_{1} \sigma_{2}+\zeta_{2} \sigma_{3}\right),
$$

$\lambda, \lambda^{\prime}= \pm 1$ and $\vec{\sigma}=\left(\sigma_{1}, \sigma_{2}, \sigma_{3}\right)$.
If two photons originate from one source their polarizations are correlated due to theorem of Landau - Yang. For decay of the scalar with $C P=+1$, their linear polarizations are parallel, for decay of the pseudoscalar with $C P=-1$, their linear polarizations are perpendicular.

$$
C P=+1
$$

$$
C P=-1
$$



For circular polarizations - directions of rotation of polarization vectors are opposite to each other (from conservation of angular momentum).

## Polarization parameters

If we have a mixture of $C P$-even and $C P$-odd states for the $h \rightarrow \gamma \gamma$, then photons can be described by a two-photon density matrix [formalism of J. Bernstein, L. Michel for $\pi^{0} \rightarrow \gamma \gamma$ ]

$$
\begin{aligned}
& \rho^{(\gamma \gamma)}=\frac{1}{4}\left(1 \otimes 1-\sigma_{3} \otimes \sigma_{3}+\xi_{1}\left(\sigma_{1} \otimes \sigma_{2}-\sigma_{2} \otimes \sigma_{1}\right)\right. \\
+\quad & \left.\xi_{2}\left(\sigma_{3} \otimes 1-1 \otimes \sigma_{3}\right)-\xi_{3}\left(\sigma_{1} \otimes \sigma_{1}+\sigma_{2} \otimes \sigma_{2}\right)\right),
\end{aligned}
$$

which is not just a direct product of two single-photon density matrices,

$$
\rho^{(\gamma \gamma)} \neq \rho^{\left(\gamma_{1}\right)} \otimes \rho^{\left(\gamma_{2}\right)}
$$

Here reference frame is chosen with the OZ axis along $\hat{\vec{k}}$, and matrices on the left (right) from $\otimes$ refer to the photon with momentum $\vec{k}(-\vec{k})$.

## Polarization parameters

Here the following parameters are introduced

$$
\begin{aligned}
& \xi_{1}=\frac{2 \operatorname{Im}\left(H_{+} H_{-}^{*}\right)}{\left|H_{+}\right|^{2}+\left|H_{-}\right|^{2}}=\frac{2 \operatorname{Re}\left(c_{\gamma} \tilde{c}_{\gamma}^{*}\right)}{\left|c_{\gamma}\right|^{2}+\left|\tilde{c}_{\gamma}\right|^{2}}, \\
& \xi_{2}=\frac{\left|H_{+}\right|^{2}-\left|H_{-}\right|^{2}}{\left|H_{+}\right|^{2}+\left|H_{-}\right|^{2}}=\frac{2 \operatorname{Im}\left(c_{\gamma} \tilde{c}_{\gamma}^{*}\right)}{\left|c_{\gamma}\right|^{2}+\left|\tilde{c}_{\gamma}\right|^{2}}, \\
& \xi_{3}=-\frac{2 \operatorname{Re}\left(H_{+} H_{-}^{*}\right)}{\left|H_{+}\right|^{2}+\left|H_{-}\right|^{2}}=\frac{\left|\tilde{c}_{\gamma}\right|^{2}-\left|c_{\gamma}\right|^{2}}{\left|c_{\gamma}\right|^{2}+\left|\tilde{c}_{\gamma}\right|^{2}} .
\end{aligned}
$$

All parameters satisfy $-1<\xi_{i}<1$.
$\xi_{2}$ defines degree of circular polarization of the photon with momentum $\vec{k}$, it has the meaning of average photon helicity, i.e.
$\xi_{2}=\langle\lambda\rangle=(+1) \frac{1}{2}\left(1+\xi_{2}\right)+(-1) \frac{1}{2}\left(1-\xi_{2}\right)$.
$\xi_{1}, \xi_{3}$ define correlation of linear polarizations of two photons.
For example, if $\tilde{c}_{\gamma}=0$ (Higgs is pure scalar in the SM), then $\xi_{1}=0, \xi_{3}=-1$ and the linear polarizations are parallel, besides $\xi_{2}=0$,
while if $c_{\gamma}=0$ (Higgs is pure pseudoscalar), then $\xi_{1}=0, \xi_{3}=1$, and they are orthogonal, also $\xi_{2}=0$.

## Amplitudes and polarizations for $h \rightarrow \gamma Z$

Helicity amplitudes for $h \rightarrow \gamma Z$

$$
H_{ \pm}=-\frac{e g m_{h}^{2}}{16 \pi^{2} v}\left(1-\frac{m_{Z}^{2}}{m_{h}^{2}}\right)\left(c_{1 Z}+c_{2 Z} \pm i \tilde{c}_{Z}\right)
$$

with decay width

$$
\Gamma(h \rightarrow \gamma Z)=\frac{1}{16 \pi m_{h}}\left(1-\frac{m_{Z}^{2}}{m_{h}^{2}}\right)\left(\left|H_{+}\right|^{2}+\left|H_{-}\right|^{2}\right) \sim\left|c_{1 Z}+c_{2 Z}\right|^{2}+\left|\tilde{c}_{Z}\right|^{2}
$$

Polarization parameters are

$$
\begin{aligned}
& \xi_{1}=-\frac{2 \operatorname{Im}\left(A_{\|} A_{\perp}^{*}\right)}{\left|A_{\|}\right|^{2}+\left|A_{\perp}\right|^{2}}=\frac{2 \operatorname{Re}\left(\left(c_{1 z}+c_{2 z}\right) \tilde{c}_{Z}^{*}\right)}{\left|c_{1 z}+c_{2 z}\right|^{2}+\left|\tilde{c}_{Z}\right|^{2}} \\
& \xi_{2}=\frac{2 \operatorname{Re}\left(A_{\|} A_{\perp}^{*}\right)}{\left|A_{\|}\right|^{2}+\left|A_{\perp}\right|^{2}}=\frac{2 \operatorname{Im}\left(\left(c_{1 z}+c_{2 z}\right) \tilde{c}_{Z}^{*}\right)}{\left|c_{1 z}+c_{2 z}\right|^{2}+\left|\tilde{c}_{Z}\right|^{2}} \\
& \xi_{3}=\frac{\left|A_{\perp}\right|^{2}-\left|A_{\|}\right|^{2}}{\left|A_{\|}\right|^{2}+\left|A_{\perp}\right|^{2}}=\frac{\left|c_{1 z}+c_{2 z}\right|^{2}-\left|\tilde{c}_{Z}\right|^{2}}{\left|c_{1 z}+c_{2 z}\right|^{2}+\left|\tilde{c}_{Z}\right|^{2}}
\end{aligned}
$$

where for further convenience we introduced

$$
A_{\|}=\left(H_{+}+H_{-}\right) / \sqrt{2}, \quad A_{\perp}=\left(H_{+}-H_{-}\right) / \sqrt{2}
$$

## How to measure circular polarization?

The circular polarization $\xi_{2}$ can be measured in the decay $h \rightarrow \gamma Z \rightarrow \gamma f \bar{f}$ :


Indeed, the angular distribution in the polar angle $\theta$ between fermion momentum in the rest frame of $Z$ boson, and direction of $Z$ momentum in the rest frame of $h$, is

$$
\begin{gathered}
\frac{1}{\Gamma} \frac{d \Gamma(h \rightarrow \gamma Z \rightarrow \gamma f \bar{f})}{d \cos \theta}=\frac{3}{8}\left(1+\cos ^{2} \theta-2 A^{(f)} \xi_{2} \cos \theta\right) \\
A^{(f)} \equiv \frac{2 g_{V}^{f} g_{A}^{f}}{\left(g_{V}^{f}\right)^{2}+\left(g_{A}^{f}\right)^{2}}
\end{gathered}
$$

with vector $g_{V}^{f}$ and axial-vector $g_{A}^{f}$ constants

$$
g_{V}^{f} \equiv t_{3 L, f}-2 Q_{f} \sin ^{2} \theta_{W}, \quad g_{A}^{f} \equiv t_{3 L, f}
$$

## Forward-backward asymmetry

Now, measurement of forward-backward asymmetry $A_{\text {FB }}$

$$
\begin{gathered}
A_{\mathrm{FB}} \equiv \frac{F-B}{F+B} \\
F \equiv \int_{0}^{1} \frac{1}{\Gamma} \frac{d \Gamma}{d \cos \theta} d \cos \theta, \quad B \equiv \int_{-1}^{0} \frac{1}{\Gamma} \frac{d \Gamma}{d \cos \theta} d \cos \theta
\end{gathered}
$$

which is

$$
A_{\mathrm{FB}}=-\frac{3}{4} A^{(f)} \xi_{2},
$$

allows one to find $\xi_{2}$.
For example, $A^{(\mu)}=0.142 \pm 0.015$, and $A^{(b)}=0.923 \pm 0.020$ [PDG2012].
This means that with $b$-quarks, asymmetry is much larger, and maybe easier to measure.

## Is it possible to measure linear polarizations?

Parameters $\xi_{1}$ and $\xi_{3}$ can be found, in principle, in the process $h \rightarrow \gamma^{*} Z \rightarrow \ell^{+} \ell^{-} Z$, with decay $Z \rightarrow \bar{f} f$ on mass shell.


Distribution over dilepton invariant mass $q^{2}$ and azimuthal angle $\phi$ between planes of decay $\gamma^{*} \rightarrow \ell^{+} \ell^{-}$and $Z \rightarrow \bar{f} f$ (in the rest frame of $h$ ):

$$
\begin{aligned}
& \frac{d \Gamma\left(h \rightarrow \ell^{+} \ell^{-} Z\right)}{d q^{2} d \phi} / \frac{d \Gamma}{d q^{2}}=\frac{1}{2 \pi}\left(1-\frac{1}{4}\left(1-F_{L}\left(q^{2}\right)\right)\right. \\
\times \quad & \left.\left(\xi_{3}\left(q^{2}\right) \cos 2 \phi+\xi_{1}\left(q^{2}\right) \sin 2 \phi\right)\right) .
\end{aligned}
$$

where

$$
F_{L}\left(q^{2}\right) \equiv \frac{\left|A_{0}\left(q^{2}\right)\right|^{2}}{\left|A_{0}\left(q^{2}\right)\right|^{2}+\left|A_{\|}\left(q^{2}\right)\right|^{2}+\left|A_{\perp}\left(q^{2}\right)\right|^{2}}
$$

is fraction of longitudinal polarization of virtual photon.

## $q^{2}$-dependent amplitudes

Amplitudes depend on dilepton invariant mass $q^{2}$ :

$$
\begin{gathered}
A_{0}\left(q^{2}\right)=\frac{e g}{16 \pi^{2} v} \sqrt{\frac{q^{2}}{m_{Z}^{2}}}\left(2 c_{1 Z} m_{Z}^{2}+c_{2 Z}\left(m_{h}^{2}-q^{2}+m_{Z}^{2}\right)\right), \\
A_{\|}\left(q^{2}\right)=-\frac{e g}{8 \sqrt{2} \pi^{2} v}\left(c_{1 Z}\left(m_{h}^{2}-q^{2}-m_{Z}^{2}\right)+c_{2 Z}\left(m_{h}^{2}+q^{2}-m_{Z}^{2}\right)\right), \\
A_{\perp}\left(q^{2}\right)=-i \frac{e g}{8 \sqrt{2} \pi^{2} v} \tilde{c}_{Z} \sqrt{\lambda\left(m_{h}^{2}, q^{2}, m_{Z}^{2}\right)},
\end{gathered}
$$

and distribution over invariant mass is

$$
\frac{d \Gamma}{d q^{2}}=\frac{\alpha_{\mathrm{em}} \sqrt{\lambda\left(m_{h}^{2}, q^{2}, m_{Z}^{2}\right)}}{48 \pi^{2} m_{h}^{3} q^{2}}\left(\left|A_{0}\left(q^{2}\right)\right|^{2}+\left|A_{\|}\left(q^{2}\right)\right|^{2}+\left|A_{\perp}\left(q^{2}\right)\right|^{2}\right)
$$

## Approximate determination of $\xi_{1}$ and $\xi_{3}$

Let us neglect longitudinal amplitude, $\left|A_{0}\left(q^{2}\right)\right|^{2} \sim q^{2}$, and $q^{2}$-dependence of transverse amplitudes. Then integrate over $q^{2}$ to obtain

$$
\begin{aligned}
& \frac{d \Gamma\left(h \rightarrow \ell^{+} \ell^{-} Z\right)}{d \phi} \approx\left(\frac{\alpha_{\mathrm{em}}}{3 \pi} \log \frac{q_{\max }^{2}}{q_{\min }^{2}}\right) \Gamma(h \rightarrow \gamma Z) \\
\times & \frac{1}{2 \pi}\left(1-\frac{1}{4}\left(\xi_{3} \cos 2 \phi+\xi_{1} \sin 2 \phi\right)\right) .
\end{aligned}
$$

where $q_{\min }^{2}$ is determined by possibilities of detectors, in particular, to provide sufficient $\phi$ resolution to separate $\sin 2 \phi$ and $\cos 2 \phi$ terms.
The accuracy of the formula improves with decreasing $q_{\text {max }}^{2}$, as the role of the competing mechanism $h \rightarrow Z^{*} Z \rightarrow \ell^{+} \ell^{-} Z$ diminishes far from $Z$-boson pole. For example, in $e^{+} e^{-}$production with invariant masses $30 \mathrm{MeV}-1 \mathrm{GeV}$, parameters $\xi_{1}, \xi_{3}$ can be determined from this formula with uncertainty about 10 - $20 \%$ (depending on a model).

## Beyond the standard model

In the $\mathrm{SM} \xi_{1}^{S M}=\xi_{2}^{S M}=0$ и $\xi_{3}^{S M}=-1$.
Deviation of $\xi_{i}$ from these values would mean presence of new physics (NP) beyond the SM.

To estimate couplings $c_{\gamma}^{N P}, c_{1 Z}^{N P}, \quad \tilde{c}_{\gamma}, c_{2 Z}, \tilde{c}_{Z}$, originating from NP, we use two models:

- the SM is effective low-energy of a fundamental (underlying) theory with a characteristic scale of NP $\wedge \gg v=246 \mathrm{GeV}$. The NP is encoded in a few higher-order operators in fields of the SM.
- Higgs interaction with fermions includes both scalar and pseudoscalar terms

where $s_{f}, p_{f}$ are real parameters and $s_{f}=p_{f}=0$ correspond to $S M$.


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To estimate couplings $c_{\gamma}^{\mathrm{NP}}, c_{1 Z}^{\mathrm{NP}}, \quad \tilde{c}_{\gamma}, c_{2 Z}, \quad \tilde{c}_{Z}$, originating from NP, we use two models:

- the SM is effective low-energy of a fundamental (underlying) theory with a characteristic scale of NP $\Lambda \gg v=246 \mathrm{GeV}$. The NP is encoded in a few higher-order operators in fields of the SM.
- Higgs interaction with fermions includes both scalar and pseudoscalar terms

$$
\mathcal{L}^{h f f}=-\sum_{f} \frac{m_{f}}{v} h \bar{\psi}_{f}\left(1+s_{f}+i p_{f} \gamma_{5}\right) \psi_{f}
$$

where $s_{f}, p_{f}$ are real parameters and $s_{f}=p_{f}=0$ correspond to SM.

## Effective field-theory approach

In effective field-theory [Manohar:2006, Buchmuller:1986, Hagiwara:1993, Grzadkowski:2010, Grojean:2013 et al.] New physics is described by gauge-invariant dimension- 6 operators 6 in fields of the SM. In the sector of Higgs and two gauge bosons $\left(\gamma \gamma, \gamma Z, Z Z, W^{+} W^{-}\right)$

$$
\begin{array}{rlr}
\mathcal{O}_{B}=i \frac{g^{\prime}}{\Lambda^{2}}\left(D_{\mu} H\right)^{\dagger}\left(D_{\nu} H\right) B^{\mu \nu}, & \mathcal{O}_{W}=i \frac{g}{\Lambda^{2}}\left(D_{\mu} H\right)^{\dagger} \tau_{k}\left(D_{\nu} H\right) W_{k}^{\mu \nu}, \\
\mathcal{O}_{B B} & =\frac{g^{\prime 2}}{2 \Lambda^{2}} H^{\dagger} H B_{\mu \nu} B^{\mu \nu}, & \mathcal{O}_{W w}=\frac{g^{2}}{2 \Lambda^{2}} H^{\dagger} H W_{k \mu \nu} W_{k}^{\mu \nu}, \\
\mathcal{O}_{W B} & =\frac{g^{\prime} g}{2 \Lambda^{2}} H^{\dagger} \tau_{k} H W_{k}^{\mu \nu} B_{\mu \nu}, & \\
\widetilde{\mathcal{O}}_{B B} & =\frac{g^{\prime 2}}{2 \Lambda^{2}} H^{\dagger} H B_{\mu \nu} \widetilde{B}^{\mu \nu}, & \widetilde{\mathcal{O}}_{w w}=\frac{g^{2}}{2 \Lambda^{2}} H^{\dagger} H W_{k \mu \nu} \widetilde{W}_{k}^{\mu \nu}, \\
\widetilde{\mathcal{O}}_{W B} & =\frac{g^{\prime} g}{2 \Lambda^{2}} H^{\dagger} \tau_{k} H W_{k}^{\mu \nu} \widetilde{B}_{\mu \nu}, &
\end{array}
$$

$g^{\prime}$ is the weak hypercharge gauge coupling, $B_{\mu \nu}$ - field strength tensor for the hypercharge gauge group, $W_{k}^{\mu \nu}$ - field strength tensor for the weak $S U(2)$ gauge group ( $k=1,2,3$ ), $H$ - Higgs doublet, $\vec{\tau}=\left(\tau_{1}, \tau_{2}, \tau_{3}\right)$. The dual field-strength tensors are $\widetilde{X}_{\mu \nu}=(1 / 2) \varepsilon_{\mu \nu \alpha \beta} X^{\alpha \beta}$, for $X=B, W_{k}$.

## Effective Hamiltonian (Lagrangian)

$$
\mathcal{H}_{\mathrm{eff}}^{(6)}=-\mathcal{L}_{\mathrm{eff}}^{(6)}=\sum_{i=B, W, B B, W W, W B} \overbrace{c_{i} \mathcal{O}_{i}}+\sum_{j=B B, W W, W B} \overbrace{\tilde{c}_{j} \widetilde{\mathcal{O}}_{j}}
$$

where $c_{i}$ and $\tilde{c}_{j}$ are effective couplings, which can be of order unity according to 'naive' dimensional analysis [A. Manohar:1984, H. Georgi:1986].
Then contribution of NP to couplings in the $h \gamma Z$ Lagrangian are

$$
\begin{gathered}
c_{1 Z}^{\mathrm{NP}}=\left(\frac{4 \pi v}{\Lambda}\right)^{2} \sin \theta_{W}\left(c_{B B} \tan \theta_{W}-c_{W W} \cot \theta_{W}+c_{W B} \cot 2 \theta_{W}\right) \\
c_{2 Z}=\left(\frac{2 \pi v}{\Lambda}\right)^{2} \frac{c_{B}-c_{W}}{\cos \theta_{W}} \\
\tilde{c}_{Z}=\left(\frac{4 \pi v}{\Lambda}\right)^{2} \sin \theta_{W}\left(\tilde{c}_{W W} \cot \theta_{W}-\tilde{c}_{B B} \tan \theta_{W}-\tilde{c}_{W B} \cot 2 \theta_{W}\right)
\end{gathered}
$$

with $\sin ^{2} \theta_{W} \approx 0.231$.
Further, if $\Lambda=4 \pi v \approx 3.1 \mathrm{TeV}$ (recall $\chi \mathrm{PT}$, where the scale $\Lambda_{\chi}=4 \pi f_{\pi}$ ), then $c_{1 Z}^{\mathrm{NP}}, c_{2 Z}, \tilde{c}_{Z}, \ldots$ are also of order unity.

## Scalar + pseudoscalar Higgs coupling to fermions

We find coefficients of NP from one-loop fermion amplitudes $h \rightarrow \gamma \gamma$ and $h \rightarrow \gamma Z$
$c_{1 Z}^{\mathrm{NP}}=-\sum_{f} N_{f} s_{f} Q_{f} g_{f} A_{1 / 2}^{Z}\left(\tau_{f}, \lambda_{f}\right)$

$$
\approx 0.3253 s_{t}-\left(8.2 s_{b}+1.2 s_{c}+0.2 s_{\tau}\right) 10^{-3}+i\left(4.8 s_{b}+0.5 s_{c}+0.1 s_{\tau}\right) 10^{-3},
$$

$\tilde{c}_{Z}=-\sum_{f} N_{f} p_{f} Q_{f} g_{f} I_{2}\left(\tau_{f}, \lambda_{f}\right)$
$\approx-0.4939 p_{t}+\left(9.6 p_{b}+1.3 p_{c}+0.3 p_{\tau}\right) 10^{-3}-i\left(4.9 p_{b}+0.5 p_{c}+0.1 p_{\tau}\right) 10^{-3}$.
and similarly for $c_{\gamma}^{\mathrm{NP}}$ and $\tilde{c}_{\gamma}$, in terms of known loop functions $A_{1 / 2}^{Z}, I_{2}$ and parameters $s_{f}, p_{f}$. Of course, for $s_{f}=p_{f}=0$ we have $c_{1 Z}^{\mathrm{NP}}=\tilde{c}_{Z}=0$.
Dominant contributions come from $t$ quark, and smaller contribution - from $c, b$ quarks and $\tau$ lepton Imaginary parts of couplings come from $c, b$ quarks and $\tau$ lepton.

## Estimate of parameters $\xi_{i}$ for decay $h \rightarrow \gamma Z$

I. Effective filed theory approach.

Tentatively $c_{B}=c_{W}=1, c_{W B}=c_{B B}=c_{W W}=1, \quad \tilde{c}_{W B}=\tilde{c}_{B B}=\tilde{c}_{W W}=1$. Choose also the NP scale $\Lambda=4 \pi v \approx 3.1 \mathrm{TeV}$.

$$
\begin{aligned}
\xi_{1} & =-0.107, \quad \xi_{2}=0.0001, \quad \xi_{3}=-0.994 \\
\mu_{\gamma Z} & \equiv \frac{\Gamma(h \rightarrow \gamma Z)}{\Gamma^{\mathrm{SM}}(h \rightarrow \gamma Z)}=1.12,
\end{aligned}
$$

Let us take a smaller scale, $\Lambda=2 \mathrm{TeV}$, then parameters deviate more from the SM values, though the circular polarization parameter is still rather small:

$$
\xi_{1}=-0.236, \quad \xi_{2}=0.0002, \quad \zeta_{3}=-0.972,
$$



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$$
\xi_{1}=-0.236, \quad \xi_{2}=0.0002, \quad \xi_{3}=-0.972, \quad \mu_{\gamma} z=1.31
$$

## Estimate of polarization parameters for decay $h \rightarrow \gamma Z$

II. Model with scalar-pseudoscalar $h f \bar{f}$ coupling.

First we need to fix parameters $p_{f}$ and $s_{f}$. Calculate the width

$$
\Gamma(h \rightarrow f \bar{f})=\frac{N_{f} G_{F}}{4 \sqrt{2} \pi} m_{f}^{2} m_{h} \beta_{f}\left(\left(1+s_{f}\right)^{2} \beta_{f}^{2}+p_{f}^{2}\right)
$$

where $\beta_{f}=\sqrt{1-4 m_{f}^{2} / m_{h}^{2}} \approx 1$ is velocity of fermion in the $h$ rest frame. Now, if

$$
\left(1+s_{f}\right)^{2}+p_{f}^{2}=1,
$$

then $\Gamma(h \rightarrow f \bar{f})=\Gamma^{S M}(h \rightarrow f \bar{f})$. Therefore the $h \rightarrow f \bar{f}$ decay widths, calculated with such values of $s_{f}, p_{f}$, agree with available CMS data for $h \rightarrow \tau^{+} \tau^{-}$and $h \rightarrow b \bar{b}$ channels.
Choose parameters satisfying $\left(1+s_{f}\right)^{2}+p_{f}^{2}=1$, e.g.

$$
p_{t}=p_{b}=p_{c}=p_{\tau}= \pm 1 / \sqrt{2}, \quad s_{t}=s_{b}=s_{c}=s_{\tau}=1 / \sqrt{2}-1
$$

and obtain polarization parameters

$$
\xi_{1}= \pm 0.121, \quad \xi_{2}=\mp 0.001, \quad \xi_{3}=-0.993, \quad \mu_{\gamma Z}=1.04 .
$$

Note that $\xi_{2}$ is again very small.

## Polarization parameters

Our estimates in two models of NP lead to conclusion that rescattering effects $h \rightarrow f \bar{f} \rightarrow \gamma Z$ on one-loop level result in values of circular polarization $\xi_{2}$ about $10^{-3}$ or smaller.

One can expect that $\xi_{2}$ remains very small in other extensions of the SM, which are CPT symmetric.

If experimental analysis of distribution for $h \rightarrow \gamma Z$ over the angle $\theta$ showed a considerable value of $\xi_{2}$, this would imply additional sources of violation of Hermiticity in an underlying (fundamental) theory at very small distances. This gives rise to additional imaginary parts of coefficients $c_{1 Z}, c_{2 Z}, \tilde{c}_{Z}$ and thereby changes value of $\xi_{2}$.

Since the Hermiticity of Hamiltonian is one of the conditions in the proof of CPT theorem, measurement of parameter $\xi_{2}$ in the decay $h \rightarrow \gamma Z \rightarrow \gamma \bar{f} f$ through forward-backward asymmetry $A_{\mathrm{FB}}$ can be useful for testing the CPT symmetry.

## Conclusions

We studied polarization observables in the Higgs boson decay $h \rightarrow \gamma Z$.

- Parameter $\xi_{2}$, which determines circular polarization of the photon, can be measured in $h \rightarrow \gamma Z \rightarrow \gamma f \bar{f}$ decay through the forward-backward asymmetry $A_{\mathrm{FB}} \sim \xi_{2}$ of the fermion $f$. This can be done either in leptonic channel $Z \rightarrow \ell^{+} \ell^{-}$, or in $b$-quark channel $Z \rightarrow b \bar{b}$, where the asymmetry may be easier to measure.
- Parameters $\xi_{1}, \xi_{3}$, related to linear polarizations of $\gamma$ and $Z$, can be approximately found from azimuthal angle distribution in cascade process $h \rightarrow \gamma^{*} Z \rightarrow \ell^{+} \ell^{-} Z$ with $Z \rightarrow \bar{f} f$ decaying on mass shell.


## Conclusions

- In any CPT symmetrical theory the circular polarization is probably very small, of the order of $10^{-3}$.
Measurement of forward-backward asymmetry in $h \rightarrow \gamma Z \rightarrow \gamma f \bar{f}$ will allow one to determine $\xi_{2}$.
- if $\xi_{2}=0 \Rightarrow$ no deviation from the SM ,
- if $\xi_{2} \neq 0 \Rightarrow$ signature of new physics, and $C P$ violation in the Higgs sector,
- if $\xi_{2} \neq 0$ and considerably larger than $10^{-3} \Rightarrow$ may indicate violation of CPT symmetry.
- Nonzero value of parameter $\xi_{1}$ will indicate violation of $C P$ symmetry in decay $h \rightarrow \gamma Z$ and thus presence of effects of new physics.

Finally, these processes may be investigated at the LHC after its upgrade to higher luminosity and energy $\sqrt{s}=14 \mathrm{TeV}$, with the integrated luminosity about $\int \mathcal{L} d t \approx 120 \mathrm{fb}^{-1}$ or bigger.

## Our publications. Acknowledgement

V.A. Kovalchuk, J. Exp. Theor. Phys. 107 (2008) 774.
A.Yu. Korchin, Talk at Int. Seminar-School "New physics and quantum chromodynamics at external conditions", Dnipropetrovsk, Ukraine, May 22-24, 2013.

Alexander Korchin and Vladimir Kovalchuk, Phys. Rev. D 88 (2013) 036009; arXiv:1303.0365 [hep-ph].

I would like to thank the Organizers for opportunity to present our results at this Conference, and kind hospitality in Ustroń!

## Thank you for attention!

## Additional slides

## Polarization parameters for decay $h \rightarrow \gamma Z$

The $h \rightarrow \tau^{+} \tau^{-}$and $h \rightarrow b \bar{b}$ decay widths, calculated with above $s_{f}, p_{f}$, agree with the CMS data [2013].
The question remains on the channel $h \rightarrow c \bar{c}$ since it is not measured yet. The $h \rightarrow c \bar{c}$ width may differ from the SM prediction, and constraint $\left(1+s_{c}\right)^{2}+p_{c}^{2}=1$ for the charm quark may not hold.
Let us assume that $\Gamma(h \rightarrow c \bar{c}) \leq \Gamma(h \rightarrow b \bar{b})$, which gives

$$
\left(1+s_{c}\right)^{2}+p_{c}^{2} \leq \mu_{b b}^{e x p} \times \frac{\Gamma^{\mathrm{SM}}(h \rightarrow b \bar{b})}{\Gamma^{\mathrm{SM}}(h \rightarrow c \bar{c})} \leq 22.8
$$

with $\mu_{b b}^{e x p}=1.15 \pm 0.62$ [CMS: 2013].
Calculation gives values $\xi_{2} \leq 8.6 \times 10^{-4}$.
Thus the existing data on Higgs decays to $\tau^{+} \tau^{-}$and $b \bar{b}$ pairs, and a reasonable assumption on the upper bound of the width to $c \bar{c}$ channel, lead to conclusion that rescattering effects $h \rightarrow f \bar{f} \rightarrow \gamma Z$ on one-loop level result in values of $\xi_{2}$ about $10^{-3}$ or smaller.

## Observability of process $h \rightarrow \gamma Z \rightarrow \gamma \ell^{+} \ell^{-}$

Let us make an estimate for the LHC, after its upgrade to energy $\sqrt{s}=14$ and higher luminosity.

For inclusive Higgs boson production in pp collisions the cross section is $\sigma_{h}=57.0163 \mathrm{pb}$ [LHC Higgs Cross Section Working Group].

Now the cross section for $p p \rightarrow h X \rightarrow \gamma Z X \rightarrow \gamma \ell^{+} \ell^{-} X$ is

$$
\sigma_{h} \times \operatorname{BR}(h \rightarrow \gamma Z) \mathrm{BR}\left(Z \rightarrow \ell^{+} \ell^{-}\right)=6.24 \mathrm{fb} \quad(\ell=e, \mu)
$$

What integrated luminosity $\int \mathcal{L} d t$ is needed to observe asymmetry?
In a hypothetical case of $\left|\xi_{2}\right|=1$ the asymmetry $\left|A_{\mathrm{FB}}\right| \approx 0.11$. The variance (error) is $\sigma_{A}=\sqrt{\left(1-A_{F B}^{2}\right) / N} \sim 1 / \sqrt{N}$, where $N$ is number of events.
To observe asymmetry on a level of 3 standard deviations,

$$
\left|A_{F B}\right|>3 \sigma_{A}, \quad N>9 / A_{F B}^{2} \approx 740
$$

With an ideal detector, this number can be obtained with $\int \mathcal{L} d t \approx 120 \mathrm{fb}^{-1}$.

## Observability of decay $h \rightarrow \gamma^{*} Z \rightarrow e^{+} e^{-} Z$ at the LHC

The process $h \rightarrow \gamma^{*} Z \rightarrow e^{+} e^{-} Z$ is very rare and is more difficult to observe.
Let us calculate cross section for $p p \rightarrow h X \rightarrow \gamma^{*} Z X \rightarrow e^{+} e^{-} Z X$ in the interval of $e^{+} e^{-}$invariant mass $30 \mathrm{MeV}<m_{e e}<1000 \mathrm{MeV}$,

$$
\sigma_{h} \times \frac{\left.\Gamma\left(h \rightarrow e^{+} e^{-} Z\right)\right|_{30<m_{\text {ee }}<1000 \mathrm{MeV}}}{\Gamma(h \rightarrow \text { all })}=0.5 \mathrm{fb} .
$$

We choose minimal $m_{e e}=30 \mathrm{MeV}$, since in recent measurements of $B^{0} \rightarrow K^{* 0} e^{+} e^{-}$branching fraction, the LHCb detector allowed selection of the lower value of $m_{e e}$ equal to 30 MeV .

When detecting $Z$ boson via $Z \rightarrow e^{+} e^{-}, \mu^{+} \mu^{-}$channels, the cross section is reduced further by 0.067 , and for $\int \mathcal{L} d t \approx 100 \mathrm{fb}^{-1}$ one can expect about 3 events.

Clearly a higher integrated luminosity will be needed to observe the decay $h \rightarrow \gamma^{*} Z \rightarrow e^{+} e^{-} Z$ and analyze its angular distribution.

